WAVES

MODULAR SYSTEM

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PHYSICS SERIES

Pitch and Loudness

The pitch (tone) of sound waves is determined by the frequency of the sound waves. Low frequency waves sound flat (low-pitched), higher frequency waves sound sharp (high-pitched).

The intensity of sound waves is interpreted as loudness (volume) of sound by the human brain. Therefore **loudness of sound is determined by the amplitude of sound waves.** A larger amplitude of oscillation is interpreted as a louder sound.

It is interesting to note that the amplitude of the faintest sound that can be heard by human ear is of the order of 10^{-11} m (~10 times smaller than the size of an atom). That is, an air molecule moves 10^{-11} m back and forth during its oscillation and the ear detects this oscillation as sound. The pressure difference produced by these oscillations is about $\Delta P \sim 3 \times 10^{-5}$ Pa. This means that for the faintest sound the human ear can detect, the air pressure is changing by this tiny amount over and under atmospheric pressure. Remembering that atmospheric pressure is around 10^5 Pa, this means that the ear is able to detect a pressure change as small as 3 parts in 10^{10} in air pressure. An analogy to this is an equal arm balance carrying two ships in equilibrium, and a feather dropped on one of the ships causes the equilibrium to be disturbed - one ship goes up - and the other goes down.

Material	Speed of sound (m/s)
Air (0°C)	331
Air (20°C)	343
Hydrogen(0°C)	1286
Water	1493
Methanol	1143
Copper	3560
Iron	5130

Table 2.1 The speed of sound in various media

Sound Level and Decibel Scale

At a frequency of 1 kHz, the intensity of the faintest sound detectible by the human ear is $I_0=10^{-12}$ W/m^2. The threshold of pain for the human ear is around 1 W/m^2. In other words, sound intensity higher than 1 W/m^2 causes pain and may permanently damage the sense of hearing.

Loudness is proportional to intensity. Since the intensity range of audible sound is so large, it is not practical to express loudness in terms of W/m^2. Instead, a logarithmic scale, the **sound level** β for a given intensity I is defined as follows:

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

The unit of β is decibel (dB). The I_0 in the formula is the reference intensity, which is chosen as the threshold of hearing, $I_0=10^{-12}$ W/m^2.

As an example, the sound level of daily conversation is about 50 dB. We can find the corresponding intensity value as follows

$$50 = 10\log\left(\frac{I}{I_0}\right)$$

$$I = 10^5 I_0 = 10^{-7}\,\frac{W}{m^2}$$

Sound source	Sound level (dB)
Jet plane (nearby)	150
Rock concert	120
Busy traffic	80
Normal conversation	50
Whisper	20
Rustling of leaves	10
Threshold of hearing	0

Table 2.2 Sound levels in decibels.

Echo

The repetition of a sound by reflection of sound waves from a surface is called an **echo.** The human brain does not process an echo if the time interval between the original sound and the echo is less than 1/15 s.

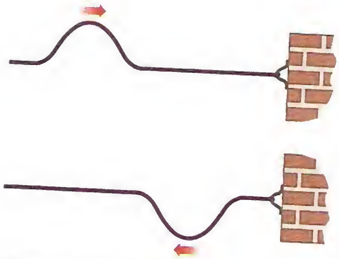


Figure 2.19 Reflection from fixed end. A 180° phase difference occurs between incident and reflected pulses.

2.6 REFLECTION AND REFRACTION

A medium in which a wave travels is characterized by the propagation speed of wave in this medium. A change of speed indicates a change of wave propagatio medium. **Whenever a wave meets a boundary between two mediums, it is p tially reflected and partially transmitted.**

Reflection

Consider a wave pulse travelling along a coil spring. One end of the spring is fi to a wall. The pulse reflects from the fixed end as shown in Figure 2.19. The refle ed pulse is inverted. In other words, in the case of reflection from a fixed end, the is a 180° phase difference between incident and reflected waves. When the pu reaches the wall, the spring applies an upward force to the wall. By the third law motion, the wall applies an equal downward force on the spring, which inverts t pulse. Actually since the wall is not infinitely rigid, a small part of the energy of t incident pulse must be transmitted to the wall. Therefore amplitudes of incident a reflected pulses are never exactly equal in reality.

A wave along a coil does not experience any phase change when it reflects from free end, as shown in Figure 2.20.

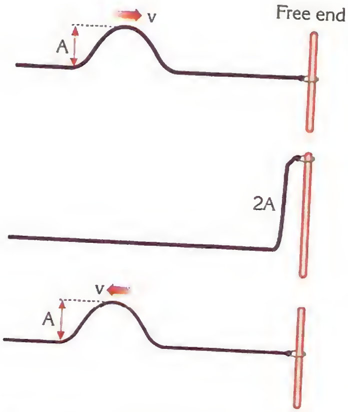


Figure 2.20 Reflection from the free end. There is no phase difference between the incident and reflected waves.

Two coil springs of different linear densities (one light and one heavy) behave as tw different mediums for waves, because the wave speed decreases with increasin spring density. Consider a wave pulse travelling along a series combination of tw springs of different linear densities. When the pulse reaches the boundary betwee different mediums, part of the energy carried by the wave is reflected back to the orig inal medium, and the remaining part is transmitted to the new medium, as shown i Figure 2.21 (a) and (b). The possible phase relationships are described in the caption.

Notice that the amplitude of the reflected wave is always smaller than the amplitude of the incident wave, because the total energy carried by the incident wave is shared between the reflected and transmitted waves.

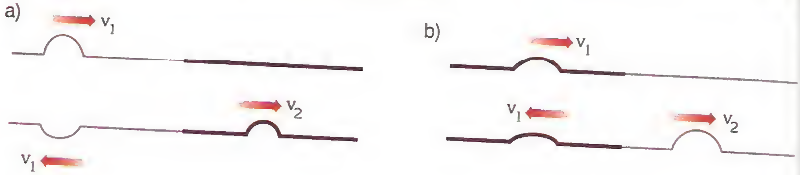


Figure 2.21 Reflection and transmission of waves on a coil spring:

a) When a pulse travelling along a light spring reflects from the boundary to a heavier spring the reflected pulse is inverted, as if it is reflecting from a fixed end.

b) When the pulse reflects from a lighter spring no inversion occurs.

In both cases the transmitted pulse does not undergo a phase change with respect to the incident pulse

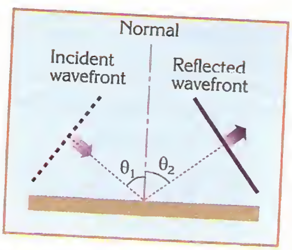


Figure 2.22 Reflection of a straight water wave from a straight barrier.

Figure 2.22 to the left shows the reflection of a straight wave pulse on a water surface from a fixed barrier. Two angles are defined.

Angle of incidence (θ_1): The angle between the incident ray and the perpendicular to the reflecting surface (called the "normal").

Angle of reflection (θ_2): The angle between the reflected ray and the normal to the reflecting surface.

Experiment shows that for two and three dimensional mechanical waves (such as surface water waves or sound waves) **the angle of reflection equals the angle of incidence.**

$$\theta_1 = \theta_2$$

This relationship is correct also for all electromagnetic waves, including light waves.

Refraction

Refraction is the bending of waves when they enter a medium where their speed is different.

When a wave passes into a new medium, its speed and wavelength change, but the frequency stays constant. This fact will now be illustrated using surface water waves as an example.

The speed of water waves decreases as waves travel from deep to shallow water. Thus, deep and shallow water behave as two different media for water waves.

Consider the ripple tank (a container having a large surface area and a flat bottom) filled with water as in Figure 2.23. Straight water waves are produced over the water surface by dipping a horizontally held long rod in the water periodically.

Figure 2.23 A Ripple tank

Suppose one side of the tank is made shallower by placing a large glass plate at one side of the bottom as shown in Figure 2.24.

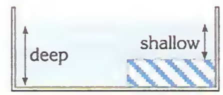


Figure 2.24 Water depth is different at each end

When periodic straight water waves produced on the deep section reach the shallow section the, speed of the waves decreases. Since the frequency stays constant, the wavelength decreases in proportion to the speed. Figure 2.25 illustrates this fact.

In general, if the wavefronts make an angle with the boundary between the mediums, the wave propagation changes direction in the new medium. Figure 2.26 shows straight wavefronts passing from medium 1 to medium 2. The waves propagate more slowly in medium 2. Notice that, first the left-most portion of the straight wavefronts pass into medium 2. The left sections of the wavefronts start to travel slower than the right sections. Thus the wave is bent.

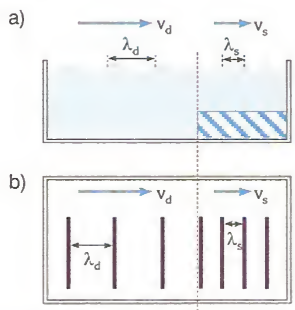


Figure 2.25 Observing wavelength change in a ripple tank a) Side view b) Top view

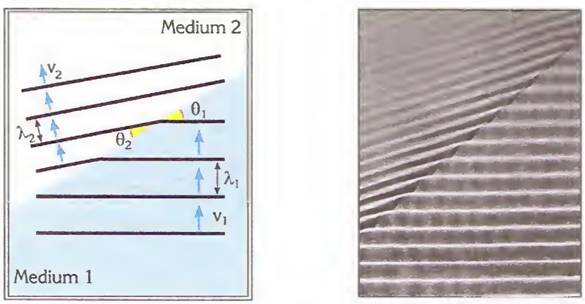


Figure 2.26 Bending of waves when passing from one medium to another. Waves are faster in Medium 1.

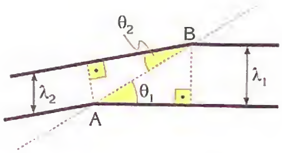


Figure 2.27 Two wavefronts refracting into a new medium

The amount of bending can be formulated as follows:

Figure 2.27 to the left shows two of the wavefronts of the refracted wave in Figure 2.26.

The two right triangles in Figure 2.27 share the same hypotenuse $|AB|$. Angles θ_1 and θ_2 are the angles the wavefronts make with the boundary. We can write

$$\frac{\text{Sin}\theta_1}{\text{Sin}\theta_2} = \frac{\lambda_1 / |AB|}{\lambda_2 / |AB|} = \frac{\lambda_1}{\lambda_2}$$

Since frequency is constant,

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1 / f}{v_2 / f} = \frac{v_1}{v_2}$$

Combining the two equations above gives the general law of refraction

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\text{Sin}\theta_1}{\text{Sin}\theta_2}$$

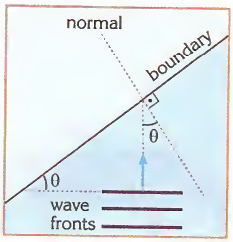


Figure 2.28 The angle between a wavefront and the boundary equals the angle between a ray and the normal to the boundary.

This relationship is applicable also to light waves as will be seen in Chapter 5.

In deriving the formula, we used the angle between a wavefront and the boundary separating the mediums. But especially when we deal with light waves in Chapter 5, it will be more convenient to use the angle between a ray and the perpendicular (called "normal") to the boundary. The refraction equation is the same in either case because the angles in question are equal. as shown in Figure 2.28.

Example 2.3 — Refraction of sound waves

A sound wave has a wavelength of 0.5 m and a velocity of 330 m/s in air. What are the frequency and wavelength of this sound wave in water where its velocity is approximately 1485 m/s?

Solution

Frequency in air is given by

$$f_{\text{in air}} = \frac{v_{\text{in air}}}{\lambda_{\text{in air}}} = \frac{330\,\text{m/s}}{0.5\,\text{m}} = 660\,\text{Hz}$$

But frequency does not change during refraction.

$$f_{\text{in water}} = f_{\text{in air}} = 660\ \text{Hz}$$

Therefore the wavelength in water is

$$\lambda_{\text{in water}} = \frac{v_{\text{in water}}}{f_{\text{in water}}} = \frac{1485\,\text{m/s}}{660\,\text{Hz}} = 2.25\,\text{m}$$

We could have used the refraction equation to get the same result

$$\lambda_{\text{in water}} = \frac{v_{\text{in water}}}{v_{\text{in air}}}\lambda_{\text{in air}}$$

$$\lambda_{\text{in water}} = \frac{1485\ \text{m/s}}{330\ \text{m/s}}\,0.5\,\text{m} = 2.25\ \text{m}$$

2.7 INTERFERENCE AND DIFFRACTION

Superposition

When two waves travelling in the same medium meet, a resultant wave is produced. According to the **superposition principle**, the displacement of every particle on the resultant wave is the sum of the displacements of the individual waves.Consider the two pulses moving in opposite directions on a string as shown in Figure 2.29. According to the superposition principle we can write

$$y = y_1 + y_2$$

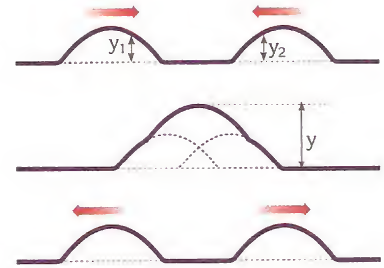


Figure 2.29 Constructive interference

Constructive interference occurs when the waves reinforce and the resultant displacement is greater than the individual displacements. Figure 2.29 shows constructive interference of two wave pulses along a string. **Destructive interference** occurs when the two waves are completely out of phase (phase difference $=180^{\circ}$). The resultant displacement is the difference in magnitude of the individual displacements. Two waves can completely cancel each other out for an instant, as shown in Figure 2.30, if waves are identical but inverted with respect to each other.

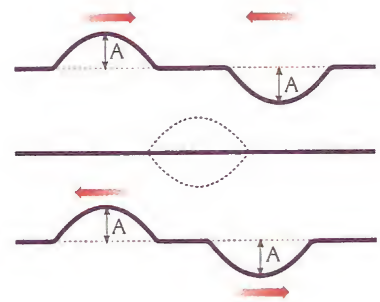


Figure 2.30 Destructive interference. The pulses completely cancel each other out if the amplitudes are equal.

Notice that in both cases, the pulses continue to move undisturbed along their original direction as if nothing happened.

Two-source Interference

Consider two identical sources producing sinusoidal waves in a medium. The sources may be two loudspeakers in air, or two separate sticks being dipped periodically in water, generating circular water waves.

The waves produced by two sources reinforce at some points in the medium. At some other points, the waves from the two sources cancel each other out. If the waves from two sources have exactly the same wavelength and amplitude, the cancellation is perfect.

Points of constructive and destructive interference are sometimes referred to as **interference maxima** and **interference minima**.

The regions of constructive and destructive interference over a water surface can easily be observed. Figure 2.31 shows such a pattern. In the regions shown by dotted red lines in the figure, the water molecules do not oscillate, due to the destructive interference of waves from the two sources. Notice that the regions of cancellation form hyperbolic lines.

Figure 2.31 Interference pattern on a water surface.

In general, for waves from two sources to cancel each other at a point, a crest from one source and a trough from the other must simultaneously reach the point. If the sources are working in phase, two crests start from the sources at the same time, but a crest and a trough meet at the point of cancellation. In other words, waves are in phase when they are produced, but when they meet at the cancellation point they are completely out of phase. The phase difference occurs because the waves travel different distances from the sources to the point of cancellation.

Figure 2.32 demonstrates how path difference results in phase difference. In all three cases in Figure 2.32, the waves start in phase from the sources but reach the given points completely out of phase. Differences in path lengths result in cancellation of waves at points indicated by dotted circles.

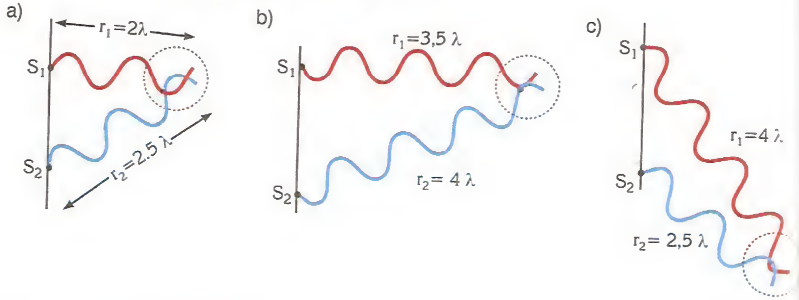


Figure 2.32 Destructive interference of waves from two sources working in phase
a) The path difference is 0.5λ b) The path difference is 0.5λ c) The path difference is 1.5λ

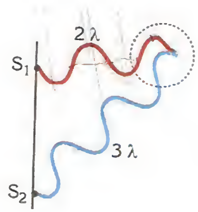


Figure 2.33 Constructive interference of waves from two sources working in phase. Path difference is λ.

Let us define the "path difference" to a point as the absolute value of the difference of path lengths from the sources to that point.

$$\text{path difference} = |r_1 - r_2|$$

where r_1 and r_2 are distances from the point to the sources.

Consider waves from two identical sources working in phase. Important cases can be formulated as follows

- Waves cancel each other out when the path difference from a point to the sources is 0.5λ, 1.5λ , 2.5λ , 3.5λ ... Figure 2.32 demonstrated this fact.
- Waves reinforce when the path difference from a point to the sources is 0, 1λ, 2λ, 3λ ... Figure 2.33 demonstrates this fact.

The condition for destructive interference and constructive interference for in-phase sources can be summarized as follows

Destructive interference	Constructive interference
$\lvert r_1 - r_2 \rvert = \left(k - \frac{1}{2}\right)\lambda$	$\lvert r_1 - r_2 \rvert = k\lambda$
$k = 1,2,3...$	$k = 0,1,2,3...$

Coherence

Two sources are **coherent** if they maintain the same phase difference between them. In other words, two harmonic wave sources are coherent if they have exactly the same constant frequency. Two coherent sources in phase stay in phase, two coherent sources out of phase stay out of phase all the time. If the phase difference between sources is not constant, the regions of constructive and destructive interference in the interference pattern continually change their positions. Therefore the

interference pattern is not stable (hence not observable) for incoherent sources.

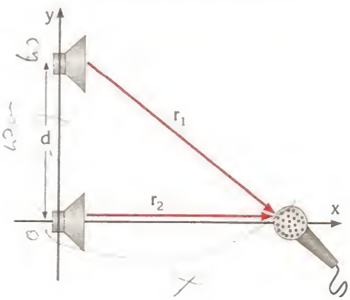
The interference phenomenon is common to all waves including sound and light waves. The interference of light waves is not easily observed, because finding two identical coherent light sources is not easy. In Chapter 5 we will see how this difficulty is overcome.

Example 2.4

Interference of sound waves

Two loudspeakers driven by the same amplifier produce sinusoidal sound waves of wavelength 20 cm. The speakers are placed along the y axis at $y_1=0$ and $y_2=40$ cm. A microphone is slowly moved along the x axis in a positive direction starting from the origin.

Find two positions along the x axis where the microphone records a minimum sound level



Solution

The microphone records a minimum intensity when the waves from two speakers cancel. The condition for destructive interference is

$$|r_1 - r_2| = \left(k - \frac{1}{2}\right)\lambda$$

$$|r_1 - r_2| = 0{,}5\lambda,\ 1{,}5\lambda,\ 2{,}5\lambda,\ \ldots$$

From the figure, $r_1 = \sqrt{d^2 + x^2}$ and $r_2 = x$

To find the first interference minimum (k=1) we use

$$|r_1 - r_2| = 0.5\lambda$$

$$\sqrt{d^2 + x^2} - x = 0.5\lambda$$

Solving for x gives

$$x = \frac{d^2}{\lambda} - \frac{\lambda}{4}$$

Inserting d=40 cm, λ=20 cm,

$$x = \frac{(40\text{ cm})^2}{20\text{ cm}} - \frac{20\text{ cm}}{4} = 75\text{ cm}$$

To find the second interference minimum (k=2) we use

$$|r_1 - r_2| = 1.5\,\lambda$$

Solving for x as before gives

$$x = 11.67\text{ cm}$$

If we try to find a third minimum using $|r_1 - r_2| = 2.5\,\lambda$ (that is, k=3), we end up with a negative distance value. Since length cannot be negative, there are only two interference minima along the x axis located at $x_1=75$ cm and $x_2=11.67$ cm.

Example 2.5

Interference pattern

Two sources work in phase on the water surface producing periodic circular waves. Explain the resulting interference pattern.

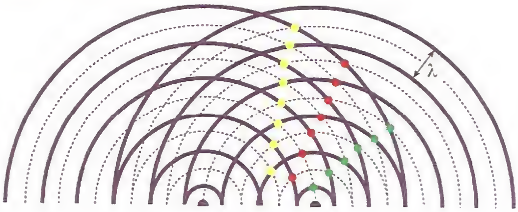
Solution

The drawing below shows the circular wavefronts produced by the sources. The solid lines represent the crests and the dotted lines represent the troughs.

The coloured dots on the diagram mark the points where a trough from one source meets a crest from the other source. Therefore, at any point along a line formed by coloured dots:

- Waves cancel each other out.
- Water molecules do not oscillate.

Examining the figure shows that



The path difference of any yellow dot is $|r_1 - r_2| = \frac{\lambda}{2}$

The path difference of any red dot is $|r_1 - r_2| = \frac{3\lambda}{2}$

The path difference of any green dot is $|r_1 - r_2| = \frac{5\lambda}{2}$

The condition for destructive interference was given by

$$|r_1 - r_2| = \left(k - \frac{1}{2}\right)\lambda$$

Therefore k=1 for all yellow dots on the drawing. Similarly, k=2 for all red dots and k=3 for all green dots on the drawing. We can conclude:

The line formed by yellow dots is the first order mir (k=1); the line formed by red dots is the second o minima (k=2); the line formed by green dots is the t order minima (k=3).

Figure 2.34 Two source interference pattern on water

Figure 2.34 shows the photograph of a two source in ference pattern on the water surface. The order of e minimum is marked on the picture. Note that each m mum has its symmetrical counterpart.

Figure 2.35 Diffraction of surface water waves

The number of minima in a two source interference pattern can be increased using a smaller wavelength or increasing the distance between sources.

Sinusoidal sound waves from two coherent speakers in air also produce an inter ence pattern in three dimensions. The regions of cancellation (minima) on suc pattern are not lines but curved *surfaces*. In Example 2.5 we located two points two such surfaces.

Diffraction

Diffraction is the bending of waves around an obstacle or spreading of waves pass through an opening. Figure 2.35 shows the diffraction of water waves though an op ing.

The amount of diffraction depends on the ratio of the wavelength "λ" to the physical of an object (or aperture) "a" as shown in Figure 2.36.

$$\text{amount of diffraction} \sim \frac{\lambda}{a}$$

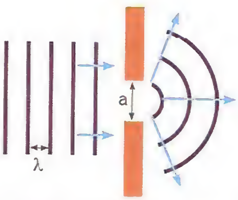


Figure 2.36 Diffraction through an opening. Amount of diffraction depends on the λ/a ratio.

When the size of a typical obstacle (or the opening size) is much greater than wavelength ($a \gg \lambda$) the effect of diffraction is negligible. Figure 2.37 demonstra this fact.

Diffraction phenomenon is common to all waves. All waves propagating in two or th dimensions (sound waves, water waves, light waves) diffract. Diffraction of light wa is difficult to observe because the wavelength of light waves is around 0.0005 m

Diffraction effects are more readily observed with sound waves which have a wavelength of the order of metres. Sound diffracts around corners or through door openings, allowing us to hear noises in other rooms. The deep (low frequency) howls of wolves spread in the forest by diffracting around trees, whereas the high frequency chirpings of small birds are contained in small areas.

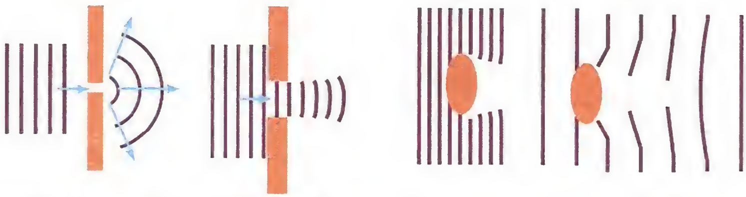


Figure 2.37 a, b) The effect of wavelength to aperture size ratio on diffraction. c, d) The effect of wavelength to object size ratio on diffraction. Notice that at (d), the the effect of an object's presence on the waves becomes invisible a few wavelengths away.

Figure 2.38 Two wavetrains of equal amplitude and frequency moving towards each other on a string

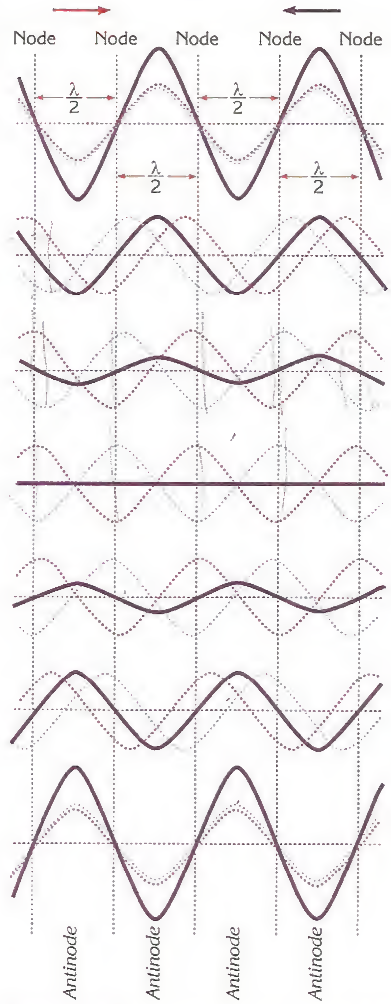


Figure 2.39 Seven snapshots of a standing wave. The solid black line shows the standing wave while the dotted lines represent the would-be positions of the two interfering waves, if they travel alone on the string.

2.8 STANDING WAVES

Standing Waves Along a Stretched String

Assume two sinusoidal waves travel in opposite directions on a string as shown in Figure 2.38. The waves have equal amplitude and wavelength. When the waves meet, they produce an interference pattern along the string.

At some points the waves interfere destructively. At points of destructive interference the resultant amplitude is zero. The zero amplitude points on the interference pattern are called **nodes**.

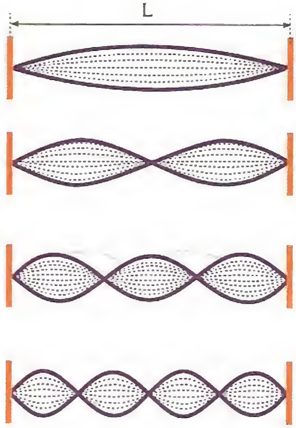
At some other points the waves interfere constructively. The points of constructive interference on the interference pattern are called **antinodes**.

If the interfering waves have exactly the same wavelength, the nodes remain fixed in position. This important fact can be justified by investigating Figure 2.39. Such an interference pattern is called a standing wave. Along a standing wave adjacent nodes are separated by a distance of $\frac{\lambda}{2}$.

Standing Waves on A String Fixed at Both Ends

Consider a long string stretched between two rigid supports. When the string is plucked, waves reflecting from the fixed ends move in opposite directions and interfere as described above. Therefore a standing wave forms on the string. Since the supports are immovable, the wave amplitude must be zero at both ends. Consequently, the fixed ends must be nodes. It is possible to set up standing waves of different wavelengths satisfying this condition. Four possibilities are shown in Figure 2.40.

The four situations shown in Figure 2.40 are called the "normal modes" or "modes" of the standing wave. As seen from Figure 2.40, each mode has its characteristic shape. When the standing wave is in one of its normal modes it has this mode's characteristic shape. In any given mode, all particles on the standing wave pass through equilibrium and reach their maximum displacement at the same instant. Therefore in any given mode, all particles on a standing wave oscillate at the same phase.



n=1 — First normal mode (First harmonic)

$$L = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2L$$

n=2 — Second normal mode (Second harmonic)

$$L=\lambda_2 \Rightarrow \lambda_2=L$$

n=3 — Third normal mode (Third harmonic)

$$L=1.5\lambda_3 \Rightarrow \lambda_3=\frac{2}{3}L$$

n=4 — Fourth normal mode (Fourth harmonic)

$$L=2\lambda_4 \Rightarrow \lambda_4=\frac{L}{2}$$

Figure 2.40 Four different possible oscillation modes of standing waves on a string fixed at both ends.

From Figure 2.40, it is apparent that, when the standing wave is in a normal mode, the length of the string between the two fixed ends must be an integer multiple of $\frac{\lambda}{2}$. This integer is generally denoted by "n". Therefore, if the length of the string is L, we can write

$$L = n\frac{\lambda_n}{2}$$

$$\lambda_n = \frac{2L}{n} \quad (n = 1,2,3,...)$$

In the equation above, n is a positive integer. Each value of n corresponds to a normal mode of the standing wave. The frequency of oscillations for the n'th normal mode is given by

$$f_n = \frac{v}{\lambda_n} = \frac{v}{2L/n}$$

$$f_n = \frac{v}{2L}n$$

Where v in the equation is the speed of interfering waves. It depends on the tension in the string and the density of the string as described in Section 2.2.

For n=1 the standing wave frequency takes its minimum value

$$f_1 = \frac{v}{2L}$$

f_1 is called the **fundamental frequency** of the oscillating system.

Accordingly, frequencies of normal modes form a harmonic series:

$$f_2 = 2f_1$$
$$f_3 = 3f_1$$
$$\vdots$$
$$f_n = nf_1$$

Frequencies of higher normal modes are integer multiples of the fundamental frequency.

- n=2 indicates the second harmonic or the first overtone as it is sometimes called.
- n=3 indicates the third harmonic or second overtone, etc...

In Section 1.9 it was stated that the natural frequency of an oscillating system is the frequency of free oscillations of that system. A given mass- spring system has a single natural frequency value. When pulled and released, the mass-spring system oscillates at that frequency.

However, a string fixed at both ends has many natural frequencies of oscillations. The frequency of each harmonic is a natural frequency for the standing wave. Therefore each normal mode has its own natural frequency of oscillations.

When pulled and released, the standing wave on the string may be in one or more of its normal modes. In other words, one or more harmonics can be excited together on a string. Figure 2.41 shows first and third harmonics excited together on a string. Which harmonics will be excited depends on how thc string is plucked.

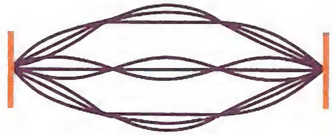


Figure 2.41 Fundamental frequency and the third harmonic excited together on a string fixed at both ends. Notice that the amplitude of the two different modes are different

Quality of Sound

The frequency of sound waves is interpreted as the pitch of sound by the brain, as stated in Section 2.5. A tuning fork produces an unpleasant, monotonous sound, because it oscillates at a single fixed frequency. The pleasant sound of musical instruments is the sum of many sinusoidal waves of different frequencies as shown in Figure 2.42.

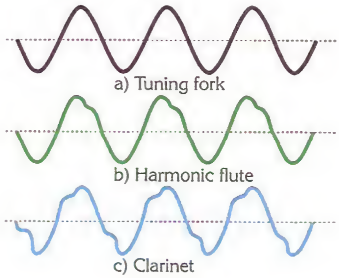


Figure 2.42 The pressure variations by three different instruments

Musical instruments such as a guitar, a piano or a violin consist of strings fixed at both ends. When a string is plucked by the musician, several harmonics with different frequencies are excited on the string, as described above. Since wave frequency is equal to source frequency, the produced sound is the sum of many waves with different frequencies.

When a violinist presses his/her fingertip on a point on the string, the effective length of the string changes. Accordingly, the frequencies of excited harmonics also change.

The fundamental frequency determines the note we hear. But the **quality** of sound is determined by the relative intensities of different harmonics.

Figure 2.43 The source sets up standing waves on the string

Wave Resonance

Consider a standing wave on a string fixed between two supports. If the supports are ideally rigid, the string is ideally elastic and all resistive forces are eliminated, the oscillation continues forever. In reality, standing waves on a string die out in time. To maintain a standing wave, energy must be continuously supplied to the string from an external source.

Suppose the right end of a string is fixed to a wall and the left end is connected to a sinusoidally oscillating source as shown in Figure 2.43. The oscillation amplitude of the source is very small compared to the string length. Therefore the left end can still be taken as a node.

In Section 1.9 it was stated that, the amplitude of a forced oscillation is a maximum value when

$$f_{external} = f_0$$

that is, when the frequency of the external driving force equals the natural frequency of oscillations. This condition is called resonance.

Wave resonance obeys the same principle. For the system shown in Figure 2.43, resonance occurs when the source frequency equals the frequency of one of the normal modes of the string.

Therefore the condition for wave resonance is

$$f_{external} = f_n$$

where f_n is the frequency of the n'th harmonic. In each resonance case a different harmonic is excited as shown in Figure 2.44.

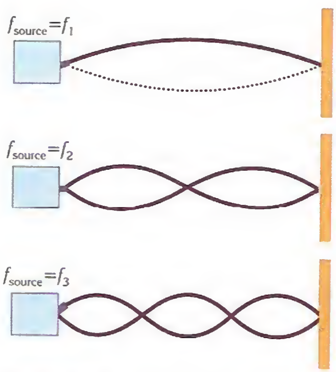


Figure 2.44 Resonance occurs when the source frequency matches the frequency of one of the harmonics of the string. Notice that at the resonance condition, wave amplitude is much bigger than source amplitude

All particles on a standing wave pass through equilibrium at the same instant. All particles on a standing wave also reach their maximum displacement at the same instant. (Figure 2.39) Therefore all particles on a standing wave oscillate at the same phase. A standing wave can be taken as a collection of many harmonic oscillators, oscillating at the same phase with different amplitudes. Amplitudes of particles may range between a negative maximum value and a positive maximum value as shown in Figure 2.45.

In Section 2.3, it was stated that two points on a travelling wave oscillate at different phases. Therefore a standing wave does not travel along the string. A standing wave is not a wave in the usual sense of the word. We call it a wave because it results from the interference of two travelling waves.

Unlike travelling waves, standing waves do not transmit energy along the string. In case of resonance between the standing wave and the wave source, energy is absorbed from the source. Since this energy is not carried away, it increases the amplitude of the standing wave. At some point, the power absorbed from the source equals the power turned into thermal energy by resistive forces like air friction. After this point amplitude stays constant.

Resonance in Air Columns

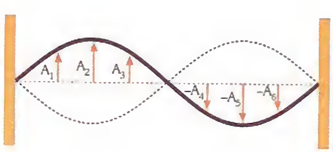


Figure 2.45 Amplitudes of particles on a standing wave range between a negative maximum value and a positive maximum value.

Musical instruments are of many types. Stringed instruments such as the violin or piano utilise stretched strings to produce sound. The operation of wind instruments such as the flute or clarinet is based on standing waves formed by sound in the instrument's body. When wind is blown down one end, sound reflecting at two ends form standing waves in the instrument. Similar to the case with string instruments, several harmonics can be excited together, depending on the competence of the musician. The effective length of the tube is changed by opening or closing the holes on the instrument body.

A wind instrument may have both ends open, or one its end closed the other end open. The molecules near the closed end have zero amplitude oscillation because

they cannot move the wall of the instrument. Therefore a closed end represents a node for displacement, as shown in Figure 2.46. Conversely, an antinode forms at the open end as shown in Figure 2.47. The normal mode's frequencies are derived in the captions.

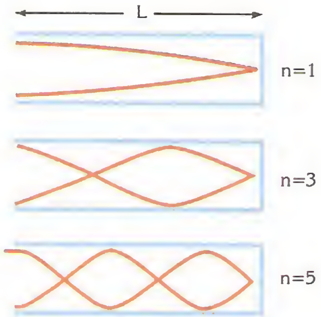


Figure 2.46 The resonance amplitude of longitudinal oscillations of air molecules in a tube open at one end, closed at the other.

Resonance wavelength-tube length relationship must be

$$\lambda_1 = 4L$$
$$\lambda_3 = \frac{4L}{3}$$
$$\lambda_5 = \frac{4L}{5}$$
$$\lambda_n = \frac{4L}{n}$$

Hence frequencies of normal modes are given by

$$f_n = \frac{v}{4L} n$$

where n=1, 3, 5, ...

because the structure does not allow any even harmonic to be excited

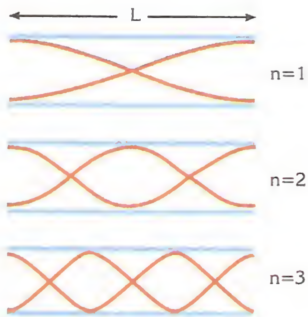


Figure 2.47 The resonance amplitude of longitudinal oscillations of air molecules in a tube open at both ends.

Resonance wavelength-tube length relationship must be

$$\lambda_1 = 2L$$
$$\lambda_2 = \frac{2L}{2}$$
$$\lambda_3 = \frac{2L}{3}$$
$$\lambda_n = \frac{2L}{n}$$

Hence frequencies of normal modes are given by

$$f_n = \frac{v}{2L} n$$

where n = 1, 2, 3, ...

Summary

A mechanical wave is the propagation of an oscillation in an elastic medium.

The motion of particles forming the medium in a **transverse wave** is perpendicular to the direction of propagation of the wave itself.

The motion of particles forming the medium in a **longitudinal wave** is parallel to the direction of propagation of the wave itself.

When a sinusoidal wave propagates in an ideally elastic medium, all the particles in the medium perform simple harmonic motion, only at different phases.

The **wavelength** of a wave is the distance between two points oscillating in phase.

The **period** of a harmonic wave is the time needed for a particle in the medium to perform one complete oscillation.

The **frequency** of a harmonic wave is the number of complete oscillations of a particle in the medium.

The **amplitude** of a mechanical wave is the maximum displacement of particles of the medium from the equilibrium position.

Mechanical waves propagate in elastic non-dispersive mediums with a speed that depends upon the characteristics of the medium.

Speed-frequency-wavelength relationship

$$v = f\lambda = \frac{\lambda}{T}$$

The energy carried by a sinusoidal wave in unit time is proportional to the square of the wave amplitude

$$\text{Power} \sim \text{Amplitude}^2$$

The equation of a wave travelling along the x axis, in the positive direction is

$$y = A \operatorname{Sin}\left[\omega\left(t - \frac{x}{v}\right)\right]$$

The phase difference between any two points on a one dimensional wave is

$$\frac{|\Delta\phi|}{2\pi} = \frac{|\Delta x|}{\lambda} = \frac{\Delta t}{T}$$

A **wavefront** is an imaginary surface joining all the points oscillating at the same phase. A **ray** is a line perpendicular to a wavefronts.

Sound waves are longitudinal waves which may propagate in solids, liquids and gases.

The **pitch** (tone) of sound waves is determined by the frequency of sound waves. **Loudness** (volume) of sound is determined by the amplitude of sound waves.

Refraction is the bending of waves when they enter a medium where their speed is different.

Refraction relationship:

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\operatorname{Sin}\theta_1}{\operatorname{Sin}\theta_2}$$

The angle of reflection equals the angle of incidence:

$$\theta_1 = \theta_2$$

The principle of superposition

$$y = y_1 + y_2$$

A **node** is a region where the waves from different sources interfere destructively. An **antinode** is a region where the waves from different sources interfere constructively.
The condition for nodes and antinodes for in-phase sources

Destructive interference	Constructive interference
$\lvert r_1 - r_2 \rvert = \left(k - \frac{1}{2}\right)\lambda$	$\lvert r_1 - r_2 \rvert = k\lambda$
$k = 1, 2, 3...$	$k = 0, 1, 2, 3...$

Diffraction is the bending of waves around an obstacle or the spreading of waves passing through an opening.

2.1 Definitions

1. What is a wave? What is the main characteristic of wave motion?

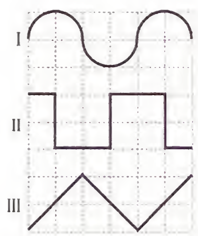
2. Define the wavelength and frequency of a wave.

3. The period of a wave is given as 0.5 s. How many wavelengths pass a given point in 5 s?

4. What is the difference between a wave pulse and a periodic wave?

5. A source produces 40 waves in 8 seconds. What are the period and frequency of the waves?

6.



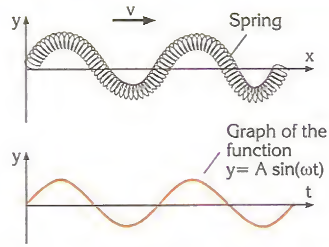
Compare the wavelengths of the waves shown in the figure.

7.

The direction of motion of a particle on a wave pulse at a given instant is shown in the figure. What is the direction of motion of the pulse?

8. What is the difference between transverse and longitudinal waves?

9.



The first figure above is the picture of a sinusoidal wave on a spring. The second figure is the graph of a sine function. The figures are similar in shape. Clearly express how the graph is related to the picture

2.2 Speed of Mechanical Waves

10. A heavy rope hangs from the ceiling. A pulse is produced at the free lower end. It is observed that the pulse gets faster as it travels up the rope. Explain why.

11. A wave has a frequency of 300 Hz and a speed of 1200 m/s. What is the wavelength of this wave?

12. The speed of waves propagating over the surface of water is 75 cm/s. What is the period and frequency of the waves if their wavelength is 5 cm?

13. A fisherman notices that the float on his fishing line oscillates 20 times in 10 seconds, and the distance between two adjacent wave crests is 90 cm. What is the speed of water waves?

14. 5 wave crests pass through a point on the water surface in 1 second. What is the wavelength, if the speed of waves is 1.2 m/s?

15. What is the frequency of radio waves having a wavelength of 300 m? The speed of radio waves in a vacuum is the speed of light, $c=3\times10^8$ m/s.

16. The distance between two adjacent wave troughs over the sea surface is 3 m. When a boat moves in the same direction as the waves, 6 waves strike its stern (rear end) in 2 seconds. If the boat moves in the direction opposite the direction of propagation of waves, 10 waves strike the stern in the same time interval. What is the speed of the boat given that the boat is slower than the waves?

17. What is the distance between a crest and the adjacent trough of a sinusoidal wave, if the wavelength is 16 cm and the amplitude is 3 cm?

18. Suppose we produce sinusoidal waves over a long, stretched spring, by sending pulses down one end. How do the frequency, speed and wavelength of the waves on the spring change if the pulses are sent at shorter intervals?

19. A sharp object is dipped into shallow water twice a second, producing harmonic waves over the water surface. The speed of the water waves is measured to be 1.1 m/s. What is the speed and wavelength of water waves if the object is dipped into the water three times a second?

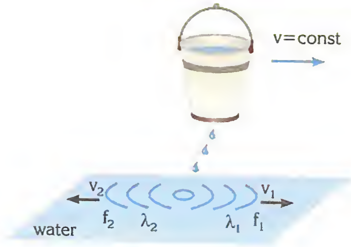
20. Which travels faster in air, the sound of a machine gun or birdsong?

21. Five wavelengths of a transverse wave pass through a point in a second. What is the frequency of oscillation of the particles in the medium?

22. The frequency range of the human voice is between (approximately) 80 Hz and 1350 Hz. What are the longest and shortest wavelengths of the human voice in air? Take the speed of sound in air as 340 m/s.

23. A pulse travelling on a string eventually disappears. What happens to the energy carried by the wave?

24.

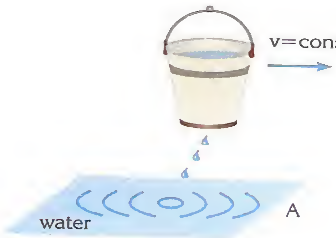


Water leaks from the bucket at a steady rate as shown in the figure. The bucket is slowly moved forward with a constant velocity over a still pond. The frequency, wavelength and velocity of the water waves moving forwards are f_1, λ_1 and v_1 and for the waves moving backwards are f_2, λ_2 and v_2.

Compare v_1 and v_2, λ_1 and λ_2, f_1 and f_2.

The speed of the bucket is less than the speed of the water waves.

25.



A bucket with a hole at the bottom hung over a ripple tank releases 4 drops of water every second. The Speed of water waves is 1.2 m/s. What is the wavelength of waves reaching side A in the figure,

a) If the bucket is stationary

b) If the bucket is moved to the right at v=0.4 m/s

2.3 The Equation of a Travelling Wave

26. What is the relationship between the wave frequency and the source frequency?

27. A piano string vibrates at a frequency of 400 Hz. Find the period of the sound waves generated.

28. A particle oscillates according to the equation: $y(t)=10\ \text{Sin}\ (5\pi t+\frac{\pi}{6})$. Find the phase of the oscillation of the particle at $t=0.1$ s

29. The equation of a travelling wave is given by $y=2\,\text{Sin}\left[8\pi\left(t-\frac{x}{5}\right)\right]$ where t is in seconds and x and y are in centimetres.

a) What is the propagation speed of the waves?

b) What is the frequency of the waves?

30. Write down the equation of a wave travelling in the +x direction with a speed of 4 cm/s. The frequency of the wave is 10 Hz and its amplitude is 3 cm.

31. A source produces harmonic waves in an elastic medium. The oscillation of a particle at x=0 is given by $y=3\text{Sin}(4\pi t)$ in cm. What is the equation of displacement for a particle at x= 50 cm, if the speed of propagation is 2 m/s?

32. The equation of a travelling wave is given by $y=4\,\text{Sin}\left[12\pi\left(t-\frac{x}{6}\right)\right]$ where t is in seconds and x and y are in centimetres. What is the wavelength of the waves?

33. Show that the wave equation $y=A\text{Sin}\left[\omega\left(t-\frac{x}{v}\right)\right]$ is equivalent to $y=A\text{Sin}\left[2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)\right]$ where ω, T and λ are the cyclic frequency, period and wavelength of the wave.

34. A wave is described by $y=4.2\,\text{Sin}\left[2\pi\left(\frac{x}{6}+\frac{t}{2}\right)\right]$, where t is in seconds and x and y are in cm.

a) What are the period and wavelength of the wave?

b) What is the velocity (speed and direction) of propagation?

35. The equation of a travelling wave is given as $y=4\,\text{Sin}\,[8\pi t-2x]$ where t is in seconds and x and y are in centimetres.

a) What is the propagation speed of the waves?

b) What is the direction of propagation (+x or –x)?

36. A wave is described by the equation $y=3\,\text{Sin}\,[32\pi t+2x]$, where x and y are in cm, and t is in s.

a) What are the speed and direction of propagation of the wave?

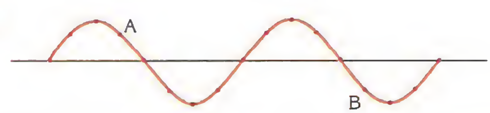
b) What are the wavelength and frequency of the wave?

37. A source produces harmonic waves in an elastic medium. The oscillation of a particle in the medium at x=0 is given by $y_s=7\text{Sin}\,(2\pi t)$ in cm. The oscillation of a particle P at a distance from the source is given by $y_P=7\,\text{Sin}\left[2\pi t-\frac{\pi}{4}\right]$. What is the distance between the source and particle P, if the speed of propagation of waves is 2 m/s?

38. The speed and frequency of a wave are 330 m/s and 660 Hz, respectively. What is the distance between two successive points having the same displacement (with respect to the equilibrium position)?

39. What is the phase difference between the first and sixth crests of a harmonic wave?

40.



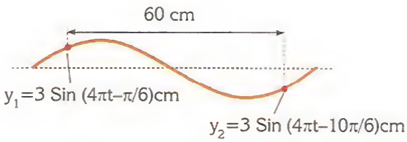
What is the phase difference between the points marked by A and B on the sinusoidal wave in the figure? The points are located at equal horizontal spacing.

41. Waves of frequency 2 Hz propagate over the surface of water at a speed of 1.2 m/s. What is the phase difference between two points lying on the same ray at a distance

a) 5 cm
b) 30 cm
c) 90 cm

from each other?

42.



The equations of instantaneous displacements of two points located 60 cm apart from each other on a travelling harmonic wave are given in the figure.

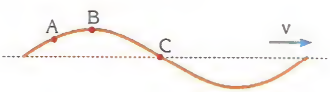
a) What is the speed of the wave?

b) What is the direction of propagation of the wave?

43. Transverse waves of frequency 5 Hz travel at a speed of 2 m/s over a very long, stretched spring. What is the phase of oscillation for a point located at a distance 60 cm away from the source, at the instant when the phase at the source is 60°?

44. The wavelength of a sinusoidal wave is 2 m. What is the distance between the closest points which oscillates completely out of phase?

45.



Three particles on a travelling sinusoidal wave at a given instant are shown in the Figure. Compare the (vertical) oscillation speeds of particles at that instant.

2.4 Propagation of Planar and Spherical Waves

46. Explain "plane wave" and "spherical wave".

47. Define "wavefront" and "ray". What is the relationship between a wavefront and a ray?

48. Explain how wave intensity changes with distance fro the source in the case of plane waves and spheric waves.

49. State two reasons why the amplitude of circular wat waves decreases with increasing distance from th source.

2.5 Sound Waves

50. Why cannot transverse mechanical waves travel in ai

51. What does the speed of sound in air depend on?

52. Which parameter of sound waves determines th loudness of sound?

53. Which parameter of sound waves determines th pitch of the sound?

54. Which insect flaps its wings faster, a fly or a mosquito

55. The blades of an airplane's propeller are invisible, a they rotate rapidly. How do we notice an increase i the rotation rate of the propeller?

56. Two microphones are located at different distance from a speaker. Which parameters - speed, frequen cy, wavelength, and amplitude - of the sound are di ferent as recorded by the two microphones?

57. How does the speed of sound in a uniform mediu change with increasing frequency? Explain.

58. Thunder is heard 10 s after a streak of lightning seen by an observer. How far away is the lightnin from the observer?

59. Ultrasound waves are used to determine sea depth. A a fixed location, waves sent from the research ship ar received 8 s later, having echoed from the sea floo What is the sea depth at this location? (See Table 2.1

60. The sound of a train is heard through the rails 2.0 s before it is heard through the air at 20°C. How far away is the train? (See Table 2.1)

61. The sound of a splash is heard 2.0 s after a stone is dropped into a well. How far below ground level is the surface of water in the well? Take $g=10\ m/s^2$ and assume free fall. (Take $v_{sound}=340\ m/s$)

62. A tuning fork vibrates at a frequency of 512 Hz. What is the wavelength of sound at 20 °C?

63. A guitar string vibrates at 200 Hz. How many oscillations does it perform whilst its sound travels 410 m in air at 20 °C?

64. The the frequency of harmonic sound waves from a loudspeaker is doubled. How do the speed and wavelength change?

65. Find the wavelength of 1000 Hz sound waves

a) in air
b) in water

Take speed of sound in water to be 1500 m/s.

66. A copper pipe is struck at one end. The sound produced is heard from the other end of the pipe twice, with an interval of 0.2 s. What is the length of the pipe? The speed of sound is 340 m/s in air and 3400 m/s in copper.

67. An observer on the ground judges by the sound that an airplane is directly overhead, at the instant she observes the plane at an angle of 60° with the horizontal. What is the speed of the plane? (Take $v_{sound}=340\ m/s$)

68. What is the intensity of a sound which has a noise level of 40 dB?

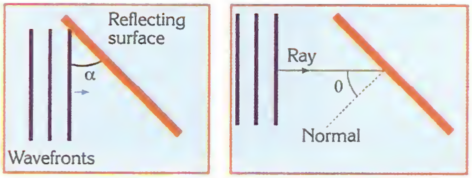
69. What is the meaning of a negative decibel value? (if there is such a thing)

70. What is the noise level (in dB) of sound which has an intensity of $10^{-6}\ W/m^2$?

71. Two bombs go off simultaneously, producing a noise level of 180 dB. What is the noise level when only one of the bombs explode?

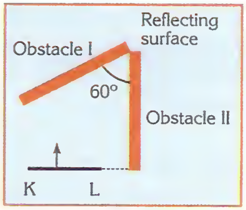
2.6 Reflection and Refraction

72.



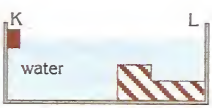
Show that the angle between plane wavefronts and a reflecting surface equals the angle between the ray and the normal to the surface. In other words, show that $\alpha=\theta$ in the figure above.

73. Show the KL pulse after it reflects from obstacles I and II, as shown in the figure.



74. Which parameters of a wave change, as the wave changes medium - speed, frequency or wavelength?

75. Which of the following quantities of a linear water wave generated at point K change, before the wave reaches point L? (Assume shallow water.)

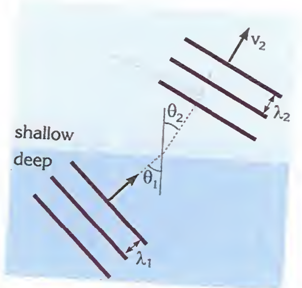


I. Period
II. Speed
III. Wavelength

76. Sound waves travel at 340 m/s in air and 3400 m/s in copper. Find the wavelength and frequency of the sound by a certain source in air, if the wavelength in copper is 3,4 m.

77. Water waves of 2 Hz frequency approach an under sea shelf (where the depth of water suddenly decreases) at a speed of 2.4 m/s. The speed of waves in the shallow section is 1.8 m/s. What are the frequency and wavelength of the waves in the shallow section?

78.

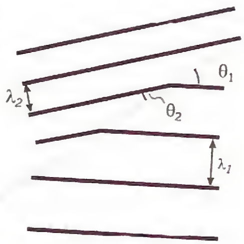


The wavefronts of water waves refracting into the shallow section of water are shown in the figure. The speed and wavelength of the waves in the deep section are 0.8 m/s and 4 cm. The speed of water waves in the shallow section is 0.6 m/s.

a) What are the wavelength and frequency of waves in the shallow section?

b) What is the angle of refraction, if the angle of incidence is 53°?

79. The wave fronts of waves passing from medium 1 to medium 2 are shown in the figure. The respective wavelengths in the two mediums are λ_1=6 cm and λ_2=5 cm.

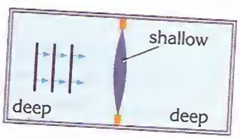


a) Draw the rays in both mediums.

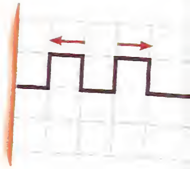
b) In which medium is the wave faster?

c) Find θ_2, if θ_1=37°.

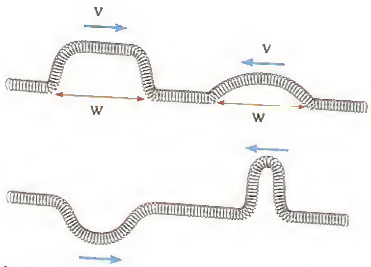
80. Draw the approximate shape of straight wavefronts on a water surface after they pass through the lens shaped shallow region.



81. Waves passing through each square in 1 s propagate as shown in the figure. How many seconds later will these waves interfere destructively?

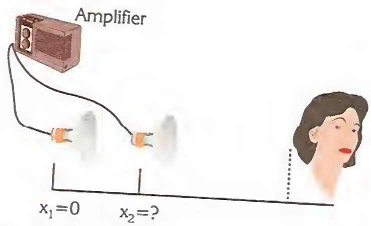


82.



Two pulses of different shape, produced on a lo[ng] spring move towards each other, as shown in the fi[g]ure. Draw the shape of the spring when the pulses mee[t].

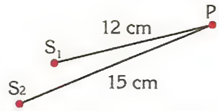
83.



The two loudspeakers produce harmonic sound waves of frequency 1000 Hz. The speed of sound in air is 320 m/s. What minimum value of x_2, is required, in cm, so that the lady doesn't hear any sound? The sources are in phase.

84. Two identical water wave sources dipping in water in phase produce an interference pattern on the surface of the water, as shown in the figure. The distances from the sources to a point on the first order (k=1) minima are measured as r_1=26 cm and r_2=25 cm. What is the wavelength of the water waves?

85. S_1 and S_2 are two sources producing identical sinusoidal waves. The sources are in phase and their wavelength is 2 cm. Is point P an interference maximum or minimum? What is its order (i.e. k=?)



86. Define diffraction. Under what conditions is the diffraction effect more significant?

87. AM radio waves (long wavelength) are better received than FM radio waves (short wavelength) if there are obstacles (buildings etc...) between the transmitter and receiver. Explain why?

88. Which sound will reach all parts (insides of caves, behind trees etc…) of a forest? The sound of a foghorn (long wavelength) or a whistle (short wavelength)?

89. Why don't we generally observe the diffraction of light waves? (Wavelength of visible light is about $5x10^{-7}$ m)

2.8 Standing Waves

90. Explain the words "node" and "antinode".

91. A travelling wave consists of particles oscillating .... phases.
A standing wave consists of particles oscillating .... phase, ... amplitude.

92. A standing wave is formed on a stretched string of 70 cm length. The string is fixed at two ends. What is the fundamental frequency of the string? The tension in the string is F=490 N and its linear density is μ=25 g/m.

93. Suppose one string of a violin is 40 cm long and has a fundamental frequency of 262 Hz. Where should the performer press her finger to produce a sound of 300 Hz frequency?

94. The lowest resonance frequency for a 2-m string fixed at both ends is 48 Hz. What is the tension in the string, if its total mass is 12 g?

95. The distance between the second and fifth nodes of a standing wave is 45 cm. What is the wavelength?

96. The fundamental frequency of a piano string is 440 Hz.
a) What is the frequency of the third harmonic?
b) What is the frequency of the third overtone?

97. Explain why the sound from a stringed instrument is sharper when the strings are made tighter.

98. A guitar string is plucked at the midpoint between two fixed ends. It is observed that only the 1^{st}, 3^{rd}, 5^{th}... etc. harmonics are excited. Explain the reason.

99. A stretched wire 1 m long has a fundamental frequency of 200 Hz. What is the speed of waves in the wire?

100. We want to increase the pitch of a sound made by a guitar string from 220 Hz to 440 Hz. By what factor must we increase the tension in the string?

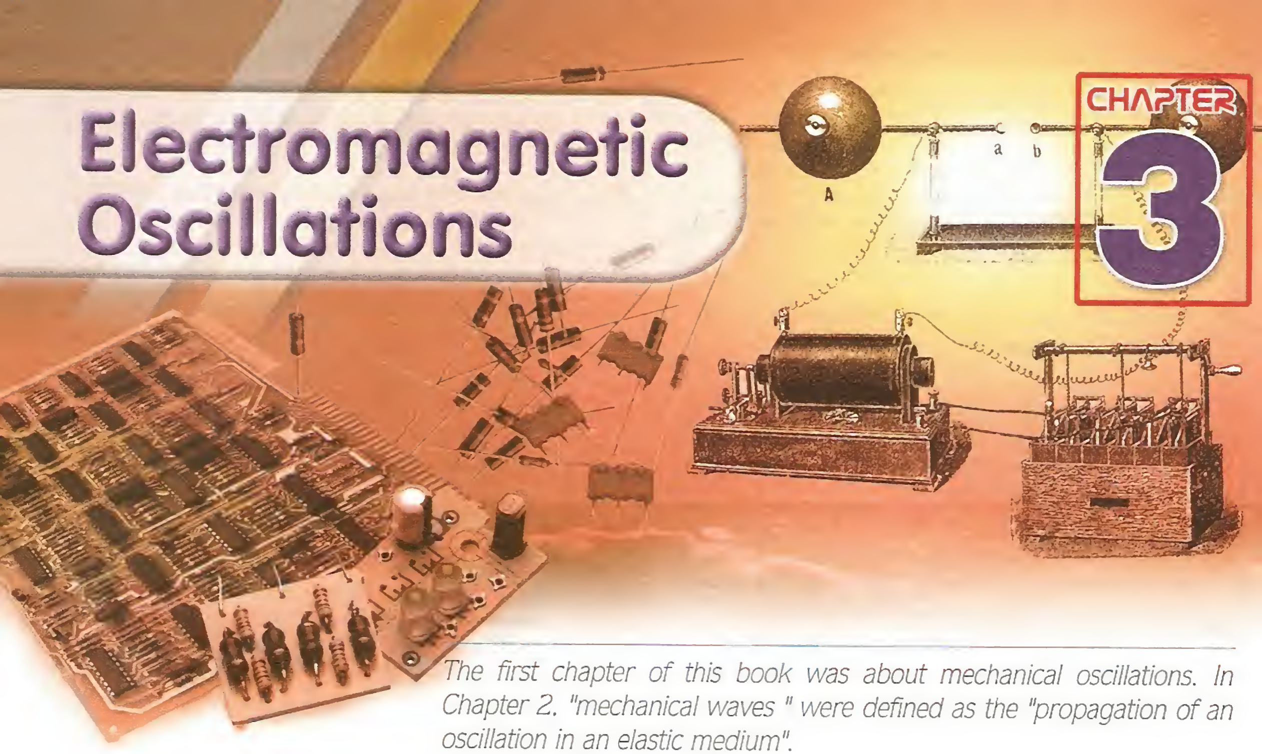
101. The three characteristics of sound are loudness of sound, pitch of sound and quality of sound. Which physical parameters of a sound wave determines each of these characteristics?

102. What is similar and what is different for the same note played by two different instruments?

103. Why do people sound different when they speak with our nose blocked, even though the vibration frequency of our vocal cords remains the same? (Think about resonant frequencies in the nasal cavity)

104. Knowing that the Martian musical notes have the same frequencies as the the notes used on Earth, a Martian musician orders a "made in Earth" flute from an online shop. Explain why he is unhappy with the sounds when he plays it on Mars.

105. Explain why a cello is much larger than a violin, considering the fact that both instruments have the same form but the sound of a violin is much higher - pitched (sharper).



CHAPTER 3

Electromagnetic Oscillations

The first chapter of this book was about mechanical oscillations. In Chapter 2, "mechanical waves " were defined as the "propagation of an oscillation in an elastic medium".

This and the next chapter have a similar relationship. This chapter is about electric charges oscillating in an electric circuit. Chapter 4 is about electromagnetic waves, which are generated by oscillating electric charges.

Some basic facts about the three basic circuit elements will now be reviewed before discussing the electromagnetic oscillations.

Figure 3.1 Two common symbols for a resistor.

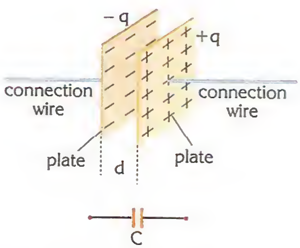


Figure 3.2 The structure of a parallel plate capacitor. When the capacitor stores an amount of charge q, the charge on the two plates are q and -q.

3.1 THREE CIRCUIT ELEMENTS

The Resistor

A resistor is a conductor (a piece of wire for example) which has resistance. The resistance of a wire is the ratio of potential difference applied across the ends of the wire to the current flowing through the wire.

$$R = \frac{U}{I}$$

The SI unit for resistance is the Ohm (Ω). Two common symbols for a resistor are shown in Figure 3.1. From the defining formula above

$$1\ \text{Ohm} = 1\frac{\text{Volt}}{\text{Ampere}} \qquad 1\Omega = 1\frac{V}{A}$$

A resistor in a circuit converts electric energy into thermal energy. In other words, a resistor heats up when current flows through it.

The Capacitor

A capacitor is designed to store electric charge. Most capacitors consist of two metal plates separated by an insulator. When the capacitor is charged, the two plates hold equal amounts of charge with opposite signs, as shown in Figure 3.2. **Capacitance** of a capacitor is defined as the ratio of charge stored in the capacitor to the potential difference across the capacitor.

$$C = \frac{q}{U}$$

The SI unit for capacitance is Farad (F).

$$1\,\text{Farad} = 1\frac{\text{Coulomb}}{\text{Volt}} \qquad 1\text{F} = 1\frac{\text{C}}{\text{V}}$$

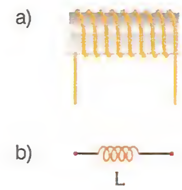


Figure 3.3 a) A solenoid is a coil of wire tightly wound around a solid core. b) Symbol of an inductor in an electric circuit

The Inductor

A coil of wire (e.g. a solenoid) resists the change of current flowing through it. This property of a coil is called **inductance**. A circuit element having inductance is called an inductor. All coils have some amount of inductance. Figure 3.3 shows a coil. Even a simple circuit (a battery, a resistor and connecting wires) can be viewed as a one-turn coil, and thus it has a small amount of inductance, which is usually neglected in calculations.

An inductor opposes the change of current through it. In other words, the inductor sets up a voltage across its terminals if the current changes. The magnitude of voltage across the inductor is proportional to the time rate of change of the current.

$$\varepsilon' = -L\frac{\Delta I}{\Delta t}$$

In the formula, ε' is the voltage across the inductor in volts, $(\Delta I/\Delta t)$ is the rate of change of current in ampere/second. L is the inductance of the inductor, which depends on the size and geometry of the device. The SI unit of inductance is Henry (H). From the formula above,

$$1\,\text{Henry} = 1\frac{\text{Volt}}{\text{Ampere/second}} = 1\frac{\text{V}}{\text{A}}\cdot\text{s} = 1\,\Omega\cdot\text{s}$$

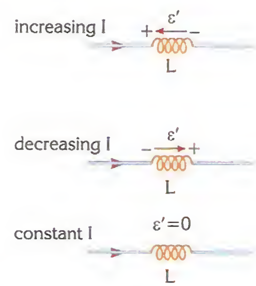


Figure 3.4 Voltage across an inductor always opposes the change of current

The minus sign in the voltage formula indicates that, the voltage across the inductor always opposes a change in the current. Figure 3.4 summarizes the three possible cases: If current through an inductor is increasing, the potential across the inductor increases in a direction opposite to the direction of the current. Conversely, if the current is decreasing, the induced potential difference tries to maintain the current. The inductor behaves like a short circuit when the current is constant.

3.2 ENERGY STORED IN CAPACITOR AND INDUCTOR

The Energy Stored in an Inductor

Consider the circuit shown in Figure 3.5. When the battery is connected in parallel with the inductor, the current starts to increase from zero. The inductor sets up a voltage in a direction to oppose the increase in the current. Hence the battery does work while driving the current against this potential difference.

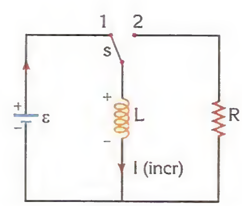


Figure 3.5 The battery does work against the potential difference across the inductor. The current passes through the inductor

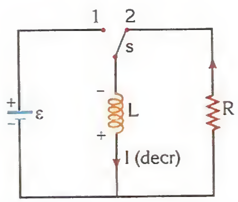


Figure 3.6 The current continues to flow even when the inductor is disconnected from the battery.

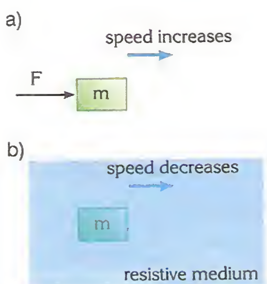


Figure 3.7 Mechanical analogy of an electric circuit in Figure 3.5 and 3.6
a) The applied force does work on the massive object increasing its speed in a frictionless medium. This process is similar to the increase of the current through an inductor because of the applied potential difference in Figure 3.5
b) The motion of the object continues even in the resistive medium owing to the inertia (mass) of the object even when the force is removed. Speed decreases because of the kinetic energy lost as heat.
The electrical counterpart of this event is depicted in Figure 3.6. The current continues to flow even in the absence of the battery, owing to the "electrical inertia" (inductance) of the inductor.

Since energy is conserved, work done by the battery against the inductor's potential difference cannot be lost. It is stored as the energy of the magnetic field produced by the inductor's coil.

We can observe this energy by throwing the switch in Figure 3.6 from position 1 to position 2. In this case the battery is removed from the circuit, and the inductor is connected in parallel with a resistor. In the instant the switch changes position, there is current in the inductor, which was driven by the battery. We observe that this current continues to flow in the same direction even without the battery, but it starts to decrease.

The current through the resistor produces thermal energy. The thermal energy produced by the resistor must have been supplied by the inductor. Since energy of the inductor is lost as thermal energy, current decreases in time. When the current drops to zero, the total heat produced by the resistor equals the initial energy stored in the inductor when it was first disconnected from the battery.

The procedure described above is very similar to a mechanical process. Suppose an object with mass m is pushed by a force F in a frictionless medium as shown in Figure 3.7a. As time passes, the speed of the object increases. This is similar to the battery increasing the current through an inductor.

When the force is removed, the object continues to move with its own inertia even against resistive forces like air friction, as shown in Figure 3.7 b. This is similar to the current' continuing to flow without the battery. The role of the mass in the mechanical process is similar to the role of inductance in the electrical process. Mass is inertia in mechanics, inductance is inertia in electricity. Mass resists a change in velocity, inductance resists a change in current.

Since friction produces heat, the speed of the object decreases in the resistive medium. When the object comes to rest, all the initial kinetic energy will have turned into thermal energy by friction. So the air friction in mechanics acts like resistance in electricity.

The formula for kinetic energy in mechanics is 1/2 mv^2. Similarly, the energy stored in the magnetic field of an inductor is given by

$$W_L = \frac{1}{2}LI^2$$

In SI units, L is in Henrys, I in Amperes, W is in Joules.

The Energy Stored in a Capacitor

Energy stored in a charged capacitor is given by

$$W_C = \frac{1}{2}qU$$

In the formulas, W is the energy stored in the capacitor in Joules, q is the charge on the capacitor in Coulombs, U is the potential difference across the capacitor in Volts and C is the capacitance in Farads.

The energy formula for a capacitor can be justified as follows:

From the definition of potential difference, work needed to move a charge q across a potential difference of U equals the product qU. The (1/2) factor in the formula comes from the fact that, while being charged, the potential difference of an initially empty capacitor **uniformly** increases from zero to the final value U.

Thus the average value of potential difference is $\frac{0+U_{max}}{2}=\frac{U_{max}}{2}$

Figure 3.8 Total mass of the boxes is m. Total work done by the man is mgh.

Figure 3.8 and 3.9 describe a similar process in mechanics. In Figure 3.9, each new box increases the height, and for identical boxes, average height is $\frac{h_{max}}{2}$.

Figure 3.9 Total mass of the identical boxes is m. Total work done by the man is mgh/2,because each new box uniformly increases the height, and the centre of mass of the vertical column is at h/2.

Using the definition of capacitance, $C=q/U$, the energy formula for a capacitor can be expressed in different forms

$$W_C=\frac{1}{2}qU=\frac{1}{2}\frac{q^2}{C}=\frac{1}{2}CU^2$$

When a charged capacitor is discharged through a resistor, as shown in Figure 3.10, it loses all its charge and the total heat generated by the resistor equals the initial energy stored in the capacitor.

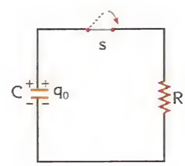


Figure 3.10 When switch s is closed, the capacitor starts to discharge through the resistor. Current through the resistance decreases in time, as the charge and potential difference of the capacitor decreases.

The table below summarises the properties of three passive circuit elements.

	R	L	C
Unit	Ohm (Ω)	Henry (H)	Farad (F)
Potential difference	IR	$-L\frac{\Delta I}{\Delta t}$	$\frac{q}{C}$
Function	Converts electrical energy into thermal energy	Stores energy in a magnetic field	Stores energy in an electric field
Energy	I^2Rt	$\frac{1}{2}LI^2$	$\frac{1}{2}\frac{q^2}{C}$

3.3 EM OSCILLATIONS IN AN LC CIRCUIT

Consider a circuit consisting of a charged capacitor and an inductor, as shown in Figure 3.11. The capacitor is initially charged and the current through the inductor is zero.

What happens, when the switch s is closed?

The capacitor starts to discharge generating a current through the inductor. The current increases as the charge in the capacitor decreases. When the capacitor is totally discharged, the current reaches its maximum value. In this case, the potential difference across the capacitor (and the inductor) is zero. However the inductor maintains the current in the same direction, by the electrical inertia it possesses. Current driven by the inductor charges the capacitor again. The current decreases as the charge in the capacitor increases. When the capacitor reaches its maximum charge once again, only in reverse polarity, the current drops to zero. Then the capacitor starts to discharge again. Thus an oscillation of charge is set up.

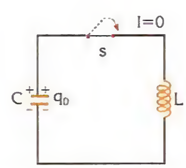


Figure 3.11 A simple oscillatory circuit consisting of a capacitor and an inductor. Initially the capacitor is charged, current is zero.

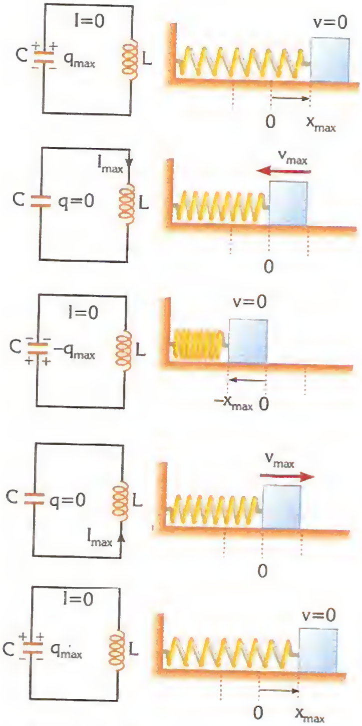


Figure 3.12 Analogy between electrical and mechanical oscillations.

Figure 3.12 shows step -by- step the process described above, drawing an analogy with the mechanical oscillations of a mass-spring system.

An oscillation taking place in an ideal (zero resistance) LC circuit without any external energy source is called a **free electromagnetic oscillation.**

Analogy Between Electrical and Mechanical Oscillations

There is a close similarity between the oscillation of charge in an LC circuit and the oscillation of a mass attached to a spring. Mathematically, the equations governing the two oscillations have the same form.

Equality of voltages across the capacitor and the inductor:	Equation of motion for the mass - spring system:
$U_{capacitor} = U_{inductor}$ $\frac{1}{C}q = -L\frac{\Delta I}{\Delta t}$	$F = ma$ $-kx = m\frac{\Delta v}{\Delta t}$

Notice that $I = \frac{\Delta q}{\Delta t}$ and $v = \frac{\Delta x}{\Delta t}$

The corresponding parameters between two oscillations are,

Electrical	Mechanical
Charge (q)	Position (x)
Current (I)	Speed (v)
Inductance(L)	Mass (m)
Inverse of capacitance 1/C	Spring constant (k)

In Chapter 1, it was shown that, if a mass spring system starts to oscillate from maximum displacement at t=0, the position and velocity of the object are given by

$$x(t)=x_{max}\,Cos(\omega t)$$

$$v(t)=-v_{max}\,Sin(\omega t)$$

where $v_{max}=\omega x_{max}$

Similarly, if one started with a charged capacitor and zero current at t=0, equations of oscillation for charge and current in an LC circuit are

$$q(t)=q_{max}Cos(\omega t)$$

$$i(t)=-I_{max}\,Sin(\omega t)$$

where $I_{max}=\omega q_{max}$. The current - time and charge-time graphs are superimposed in Figure 3.13. Since $(-\sin\omega t) = \cos(\omega t+\pi/2)$, current is 90° ahead of voltage.

* In derivative notation, the two formulas relating charge and current can be written as $q/C=-LI'$ and $I=q'$. Combining these two formulas yields $q''=-\frac{1}{LC}q$, which is a "differential equation". Its solution is an oscillation.

Frequency of Oscillations

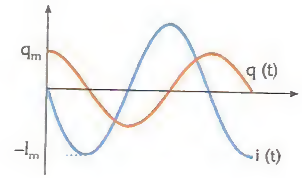


Figure 3.13 The time dependence of charge and current in free LC oscillations.

The angular frequency of free mechanical oscillations are given by

$$\omega = \sqrt{\frac{k}{m}}$$

Using the analogy between the mechanical and electromagnetic oscillations, the angular frequency of LC oscillations can be determined. Replacing k by 1/C and m by L gives

$$\omega = \sqrt{\frac{1}{LC}}$$

Thus the frequency and period of free electromagnetic oscillations in an LC circuit are given by

$$f = 2\pi\sqrt{LC} \quad \text{and} \quad T = \frac{1}{2\pi\sqrt{LC}}$$

3.4 CONSERVATION OF ENERGY

In an ideal LC circuit, resistance is zero. Thus heat is not produced during electromagnetic oscillations. In Chapter 4, it will be shown that oscillating electric charges produce electromagnetic (EM) waves. Therefore, even in the absence of resistance, the total energy of oscillations decreases in time because of the energy radiated as EM waves. But the energy lost to EM wave generation in a few cycles is generally small compared to the total energy of the system. Therefore it is possible to take the total energy of oscillations as constant during one cycle.

In other words, we can assume that the total energy of free electromagnetic oscillations in an LC circuit is constant.

In Chapter 1 it was shown that in free mechanical oscillations, total energy is also constant. At maximum displacement, when the oscillating object momentarily stops, all the mechanical energy is in the form of potential energy stored in the spring. As the oscillating object passes through equilibrium, kinetic energy takes its maximum value because potential energy is zero.

Similarly, in electromagnetic oscillations, the total energy is alternately stored in the electric field of the capacitor and the magnetic field of the inductor. When the current is zero, charge stored in the capacitor takes its maximum value, therefore all the energy is stored in the capacitor's electric field. The capacitor is totally discharged when the current reaches its maximum value. In this case all the energy is stored in the inductor's magnetic field.

Since the total energy is constant, the maximum energy stored in the capacitor must be equal to the maximum energy stored in the inductor, since when energy stored in one form reaches its maximum value, the energy stored in the other form is zero, as described above. In the intermediate steps the energy is shared between the energies of the electric and magnetic fields, its total value always remaining constant.

Therefore the conservation of energy for free LC oscillations require

$$W_{total} = \text{constant}$$

$$W_{total} = \frac{1}{2}\frac{q_{max}^2}{C} = \frac{1}{2}LI_{max}^2 = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}LI^2$$

Example 3.1

A 1 μF capacitor is charged up to a 100 V potential and is then placed in parallel to an inductor of inductance L=1 H. Express charge and potential difference as functions of time.

Solution

The angular frequency of oscillations is given by

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10^{-6}F)(1H)}} = 1000 \text{ rad/s}$$

Maximum charge on the capacitor is given by

$$q_{max} = CU_{max} - (10^{-6}F)(100V) = 10^{-4}C$$

The oscillation starts with maximum charge on the capacitor, that is, $q = q_{max}$ at t=0. Therefore the time dependence is given by

$$q(t) = q_{max}\text{Cos}(\omega t)$$

Inserting the calculated values

$$q(t) = (10^{-4})\cdot\text{Cos}(1000t) \text{ in Coulombs}$$

The potential difference across the capacitor is always proportional to its charge. Therefore charge and potential difference change in phase:

$$U(t) = U_{max}\text{Cos}(\omega t)$$

Inserting the calculated values,

$$U(t) = (100)\cdot\text{Cos}(1000t) \text{ in Volts}$$

Example 3.2

Energy conservation

The maximum values of charge and current are measured as $q_m = 10^{-6}$ C and I_m=2mA in an ideal LC circuit. Find the frequency of oscillations.

Solution

From energy conservation

$$\frac{1}{2}\frac{q_m^2}{C} = \frac{1}{2}LI_m^2$$

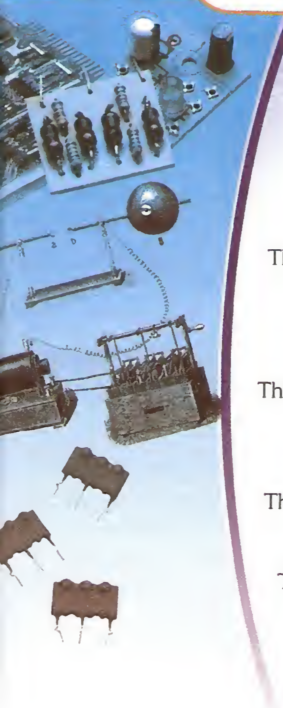
$$\frac{1}{LC} = \frac{I_m^2}{q_m^2}$$

$$\frac{1}{\sqrt{LC}} = \frac{I_m}{q_m}$$

$$\omega = \frac{I_m}{q_m} = \frac{2\cdot 10^{-3}A}{10^{-6}C}$$

$$\omega = 2000 \text{ rad/s}$$

Summary



Analogy between electrical and mechanical oscillations

Electrical	Mechanical
Charge (q)	Position (x)
Current (I)	Speed (v)
Inductance(L)	Mass (m)
Inverse of capacitance 1/C	Spring constant (k)

The time dependence of charge and current in free oscillations

$$q(t)=q_{max}Cos(\omega t)$$

$$i(t)=-I_{max}\,Sin(\omega t)$$

The frequency of oscillations

$$\omega=\sqrt{\frac{1}{LC}}$$

The relationship between the maximum values of charge and current

$$I_{max}=\omega q_{max}$$

The conservation of energy in free LC oscillations

$$W_{total}-const$$

$$W_{total}=\frac{1}{2}\frac{q_{max}^2}{C}=\frac{1}{2}LI_{max}^2=\frac{1}{2}\frac{q^2}{C}+\frac{1}{2}LI^2$$

QUESTIONS AND PROBLEMS

3.1 Three Circuit Elements

1. Express Farad and Henry in terms of Ohms and seconds.

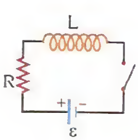
2. Express Ohm, Farad and Henry in terms of the basic SI quantities: ampere, second, kilogram and the metre.

3. The charge on a capacitor is doubled. By what factor does the capacitance change?

4. A steady current of 5 A flows through a 0.5 H inductor for 10 s. What is the potential difference across the inductor?

5. Explain how the direction of self-induced emf is determined in an inductor

6. Which side of the inductor is at higher potential when the switch is first closed?



7. A 100 μF capacitor is charged up to a potential difference of 20 V.

a) What is the energy stored in the capacitor?

b) What is the charge of the capacitor?

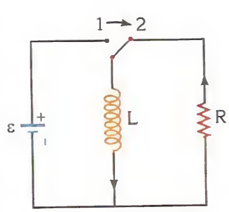
8. Show that the formulas for the energy stored in a capacitor $\frac{1}{2}qU$, $\frac{1}{2}\frac{q^2}{C}$ and $\frac{1}{2}CU^2$ are equivalent.

9. A charge of 10^{-5} C is stored in a capacitor produces a 200 V potential difference. What is the energy of the capacitor?

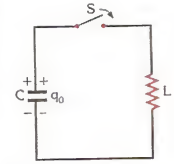
10. What is the energy stored in the magnetic field of a 20 mH inductor, if it carries a 0.5 A current?

11. A 20 mH ideal inductor is connected in parallel with an ideal battery (with zero internal resistance). What is the work done by the battery in the interval during which the current increases from 2 A to 8 A?

12. In the circuit diagram shown, after the switch is thrown from position 1 to position 2, the current through the 0.5 H inductor gradually decreases to zero. How much heat is produced in the resistor, while the current decreases from 10 A to 4 A?



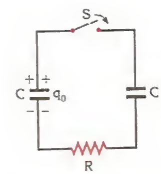
13. A capacitor having an initial charge of 20 mC is discharged through a resistor as shown in the figure. The capacitance and resistance values are given as C=400 μF and R= 50 Ω.



a) What is the current through the resistor at the instant when the charge on the capacitor is 5 mC?

b) How much heat is released by the resistor up until that time?

14.



A capacitor having a capacitance of 5 mF is charged up to a potential difference of U=20 V and connected to an initially empty, identical capacitor through a resistor.

a) What is the initial charge on the capacitor?

b) What are the final charges on the capacitors?

c) How much heat is released from the resistor?

3.3 EM Oscillations in an LC Circuit

15. The time dependence of the charge on the capacitor in an LC circuit is given by $q(t)=0.1\cos(10^4\pi t)$ in μC. ($1\mu C=10^{-6}$ C)

a) What is the frequency of oscillations?

b) What is the maximum charge on the capacitor?

16. Charge oscillates in an ideal LC circuit by the equation; $q(t)=0.4$ Cos $(300t)$ in mC. What is the amplitude of the charge oscillation?

17. The oscillation of charge in an LC circuit is given by $q(t)=(5nC)\text{Cos}(1000\,\pi t)$. What are the initial values of charge and current? (1 nC = 10^{-9} C)

18. The instantaneous value of charge on the capacitor in an LC circuit is given by $q(t)=(40\mu C)\cos(10^5\pi t)$. The capacitance is 20 μF.

a) What is the maximum value of the voltage across the capacitor?

b) Express the potential difference across the capacitor as a function of time, $U(t)=?$

19. Charge oscillates in an ideal LC circuit by the equation; $q(t)=0{,}3\cos(300t)$ in mC. What is the equation for the instantaneous value of current, $i(t)=?$

20. An LC circuit has a 1 μF capacitor and a 1 H inductor. What is the cyclic frequency of the EM oscillations?

21. An LC circuit is designed to have a natural cyclic frequency of 10 MHz. What is the inductance of the coil, if a 25 pF capacitor is used in the circuit? (1 pF = 10^{-12} F)

22. An LC circuit has variable capacitance and inductance. The inductance of the circuit may range between $1\mu H$ and $1 mH$. The capacitance may vary between the limits $1pF$–$10\mu F$. What is the range of cyclic frequencies that can be generated by this circuit?

23. An ideal LC circuit consists of a 50 μF capacitor and a 2 H inductor. At $t=0$, the potential difference across the capacitor is 20 V and the current is zero. Write the time dependence of

a) voltage, $U(t)=?$

b) charge, $q(t)=?$

c) current, $i(t)=?$

24. A $5\mu F$ capacitor is charged up to a 100 V potential and connected to a 0.2 H inductor. Assume energy is not lost.

a) What is the potential across the capacitor $t=0.01\pi$ s after the capacitor starts to discharge?

b) What is the current in the circuit at a time $t=0.01\pi$ s after the capacitor starts to discharge?

25. The maximum value of charge in the capacitor of an oscillatory circuit is 100 μC. The frequency of oscillations is 2 kHz. The oscillation starts with a full capacitor, that is, at $t=0$, $q=q_{max}$. When, for the first time, does the charge on the capacitor take the value of 86.7 μC?

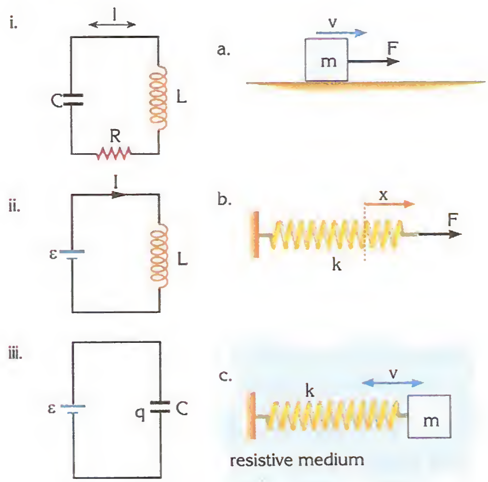
26. By what factor does the natural frequency of EM oscillations in an ideal LC circuit change when the inductance is increased by a factor of four?

27. The frequency of oscillations decreases by a factor of 4 in an LC circuit when the capacitance of the circuit is increased by 0.15 μF. What is the initial capacitance? (Given L=constant)

28. By what factor does the frequency of EM oscillations in an ideal LC circuit change, when a dielectric with dielectric constant of $\varepsilon=4$ is inserted between the plates of the capacitor?

29. The distance between the plates of a capacitor in an LC circuit is increased by a factor of 9. By what factor does the frequency of EM oscillations increase or decrease?

30. Match each of the electrical circuits with its corresponding mechanical analog.



31. Show that the cyclic frequency of free oscillations in an LC circuit is given $\omega = \frac{I_{max}}{q_{max}}$ by using the conservation of energy.

32. In a harmonic LC oscillation the maximum energy stored in the electric field is 2 J and the maximum energy stored in the magnetic field is 2 J. What is the total energy of the oscillations?

33. The maximum energy stored in the capacitor during electromagnetic oscillations in an LC circuit is 0.01 J.

a) What is the maximum energy in the inductor?

b) What is the total energy in the circuit?

34. The maximum values of charge and current are measured as q_m=2 μC and I_m=6 mA in an LC circuit. What is the cyclic frequency of oscillations? Assume there is no heat loss.

35. What is the ratio of energy stored in the inductor to the energy stored in the capacitor when 1/3 of the total energy is stored in the capacitor of an LC circuit?

36. An LC circuit consists of a 100 μF capacitor and a 0.1 mH inductor. What is the maximum value of charge in the capacitor, if the maximum current through the inductor is 2 mA?

37. A 100 μF capacitor is charged up to a potential difference of 5 V and connected in parallel to a 10 mH inductor to set up electromagnetic oscillations. What is the current through the circuit when the instantaneous value of potential difference across the inductor is 3 V?

38. A 1 μF capacitor is charged up to a 50 V potential difference and then connected to a 1 H inductor. What is the energy stored in the capacitor at $t=10^{-3}\pi$ s after the capacitor starts to discharge?

39 An LC circuit consists of a 400 μF capacitor and a 0.01 H inductor. The maximum charge on the capacitor is 16 mC.

a) What is the total EM energy in the circuit?

b) What is the ratio of instantaneous energy of the capacitor to that of the inductor when the instantaneous charge in the capacitor is 1 mC?

40. An LC circuit consists of a 200 μF capacitor and an 8 mH inductor. The maximum value of current through the inductor is 50 mA.

a) What is the maximum value of energy stored in the capacitor?

b) What is the charge in the capacitor when the instantaneous current through the inductor is 10 mA?

41. The capacitance of an ideal LC circuit is 2 mF and the maximum voltage across the capacitor is 20V. Find the energies stored in the electric and magnetic fields, when the voltage across the capacitor is half of its maximum value during the oscillation.

42. In an ideal LC circuit, an increase of maximum potential difference by 10 V across the capacitor causes the maximum current to be doubled. What is the initial potential difference of the capacitor?

43. Maximum values of charge stored in the capacitor and current through the inductor in an LC circuit are 4 μC and 1mA, respectively. What are the instantaneous values of charge and current, when the instantaneous energy of the inductor equals that of the capacitor?

44. A 200 μF capacitor is charged up to a potential difference of 20 V and connected to an inductor to form an LC circuit. What are the energies of the electric field in the capacitor and the magnetic field in the inductor when the instantaneous value of potential difference is 8 V?

45. What is the ratio of the energy stored in the capacitor to the energy stored in the inductor in an ideal LC oscillation,

a) At t=T/4 after the capacitor starts to discharge?

b) At t=T/8 after the capacitor starts to discharge?

46. The maximum value of the current through the inductor in an LC circuit decreases by 20 mA in a given time, because of the heat loss during EM oscillations. It is observed that the maximum charge in the capacitor drops to one-third of its initial value in the same time. What is the initial value of maximum current through the inductor?

47. The total energy of electromagnetic oscillations in an LC circuit decreases in time due to the resistance of the wires. By which factors do the values of the following parameters change, when the maximum value of charge in the capacitor decreases to half its initial value?

a) Potential difference across the capacitor

b) Maximum current through the inductor

c) Maximum value of total EM energy in the circuit.

48. A 10 μF capacitor is charged up to a 100 V potential and connected to an inductor. The maximum potential decreases in time due to the small resistance in the wires. Find the heat produced in the circuit up until the maximum potential drops to 40 V.

49. An ideal LC circuit consists of an inductor of 40 mH inductance and a capacitor of 1 μF capacitance. The maximum voltage across the capacitor is $U_{max}=10$ V. Given that at t=0, the voltage across the capacitor is 6V, and the capacitor is being charged.

a) Express the time dependence of the voltage $U(t)=?$

b) What is the value of current at t=0?

Electromagnetic Waves

Radio waves, microwaves, infrared rays, ultraviolet rays and visible rays, X-rays and gamma rays are all types of electromagnetic waves. Though different types of EM waves have different effects, all EM waves share certain common characteristics. The reflection, refraction, interference and diffraction of mechanical waves were all discussed in Chapter 2. In a similar fashion, EM waves can also undergo reflection, refraction, interference and diffraction.

Figure 4.1 Electric field of a stationary electric charge

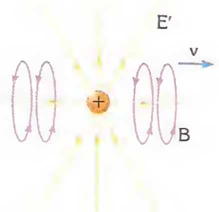


Figure 4.2 A uniformly moving electric charge produces a magnetic field in addition to its electric field. The electric field of a moving charge is different to the electric field of a stationary charge.

4.1 ELECTRIC AND MAGNETIC FIELDS

A stationary electric charge has its electric field, filling all space, represented by electric field lines as in Figure 4.1. A uniformly moving electric charge generates a magnetic field in addition to its electric field, as shown in Figure 4.2. The electric field of a moving charge is different to the electric field of the same charge when it is stationary.

The Induced Electric Field

Consider a stationary loop of wire towards which a magnet is moved as shown in Figure 4.3. According to Faraday's law of induction, a changing magnetic flux induces current in a loop of wire. The force acting upon the free electrons in the wire cannot be magnetic in this case, since the loop is stationary. A magnetic field does not act on stationary charges.

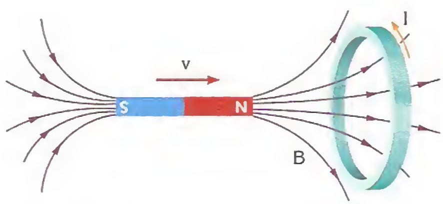


Figure 4.3 A changing magnetic flux induces current in a loop of wire

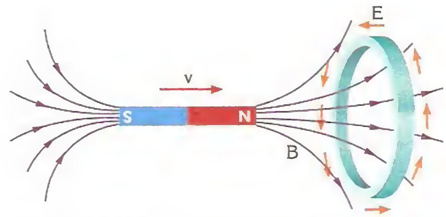


Figure 4.4 A current is caused by the electric field generated by the changing magnetic field

This means that the force maintaining the current in the wire must be of an electrical nature. This short analysis shows that the change of magnetic field through the loop of wire produces an electric field around the loop, as shown in Figure 4.4.

In general, **a changing magnetic field produces an electric field**, as shown in Figure 4.5. The magnitude of the electric field is proportional to the rate of change of the magnetic field.

$$E \sim \frac{\Delta B}{\Delta t}$$

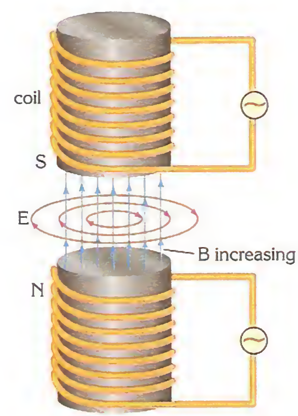


Figure 4.5 A changing magnetic field produces an electric field

The Induced Magnetic Field

An electric current generates a magnetic field. Now consider a simple circuit consisting of a battery, an initially uncharged capacitor, connecting wires and a switch, as shown in Figure 4.6. When the switch is first closed, the current set up in the wires starts to charge the capacitor. While the capacitor is charged, the electric field in the capacitor increases together with the capacitor's charge .

During the charging process, the current in the wires generates a magnetic field around the wires as shown in Figure 4.6. However, there is no current through the capacitor, since the electrons cannot jump from one plate to the other. James Clark Maxwell (1831-1879) proposed that, although the current is zero through the capacitor, the changing electric field in the capacitor generates a magnetic field, as if there were a current.

In general, **a changing electric field produces a magnetic field.** The magnitude of the magnetic field is proportional to the rate of change of the electric field.

$$B \sim \frac{\Delta E}{\Delta t}$$

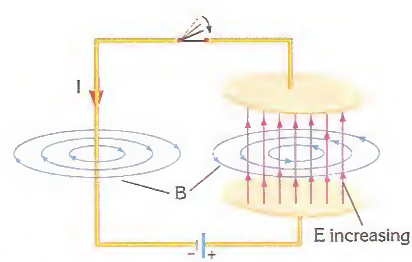


Figure 4.6 A changing electric field produces magnetic field.

4.2 ELECTROMAGNETIC WAVE

Using the symmetry between the electric and magnetic fields (a change in one of the fields produces the other type of field), Maxwell proposed that changing electric and magnetic fields can propagate in space by reproducing each other. Therefore electromagnetic waves must exist.

Electromagnetic waves consist of changing (oscillating) electric and magnetic fields that reproduce each other and propagate in free space without requiring a medium.

Maxwell showed that, theoretically, the speed of these waves is given by

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

where ε_0 is the permittivity of free space and μ_0 is the permeability of free space. Placing the numeric values in the formula gives

$$v = \frac{1}{\sqrt{(8.85\times10^{-12}\ \text{F/m})\cdot(4\pi\times10^{-7}\ \text{Tm/A})}} \cong 3\times10^{8}\ \text{m/s}$$

The theoretically obtained value for the speed of electromagnetic waves was equal to the measured speed of light. Thus Maxwell concluded that light is an electromagnetic wave. The speed of EM waves in vacuum is denoted by the letter "c".

$$c = 299\,792\,458\ \text{m/s} \cong 3\times10^{8}\ \text{m/s}$$

The Source of EM Waves

Electromagnetic waves are produced by accelerating electric charges.

A uniformly moving electric charge drags along its electric and magnetic fields as shown in Figure 4.2. An **acceleration** of the charge causes a sudden change in the charge's electromagnetic field; the field breaks away from the charge and starts to propagate as an independent entity: An electromagnetic wave.

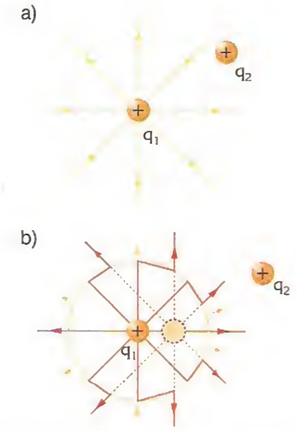


Figure 4.7 a) Two charges interact by means of an electric field. For clarity, the electric field of one of the charges is not shown.
b) The sudden acceleration of q_1 causes a ripple in the electric field; q_2 does not "know" about this acceleration yet

Consider two stationary electric charges situated at a distance from each other, as shown in Figure 4.7. The Coulomb force acts on the charges. Now suppose one of the charges is given a sudden displacement. The second charge will also be affected by this change, but not instantaneously. The "information" of change in the situation of the first charge travels at the speed of light, in the form of an EM wave. In other words, sudden acceleration of the first electric charge causes a "ripple" in its electric field, which previously pervaded all space. The ripple in the electric field - along with the accompanying magnetic field- propagates in space with a finite speed; the speed of light. When the "ripple" reaches the position of the second charge, the charge experiences a change in the force acting on it.

EM waves can be generated by simply waving a charged ebonite rod (or a plastic pencil) around. Waving the rod 5 times per second produces EM waves of frequency 5 Hz. Such a low frequency wave carries very little energy and is very difficult to detect. Typically the frequency of a radio wave lies between 100-1000 MegaHertz. To oscillate charges at such high frequencies, electronic circuits are used.

Charges *oscillating* along a conductor generate electromagnetic waves, since oscillation implies acceleration. The electric field in the immediate vicinity of a periodically oscillating charge also changes periodically. The period of changes in the field equals the period of oscillations. The changing electric field generates a changing magnetic field, which in turn generates a changing electric field at increasing distances from the source.

The EM waves from a sinusoidally oscillating source also have an sinusoidal character. Nevertheless, all EM waves do not have to be of sinusoidal (or even oscillatory) character. An electric charge that suddenly starts or stops also experiences acceleration and produces a short pulse of an EM wave which is not in the form of a sinusoidal wave.

Properties of EM Waves

The behaviour of electromagnetic field near its source is quite complicated. The following description pertains to EM waves which are several wavelengths away from a harmonically oscillating source, such as electrons oscillating along a long conductor (called an antenna).

At large distances from their source, spherical waves travelling in three dimensions become nearly planar. The wavefronts can be taken as planes, as described in Section 2.4.

Electromagnetic waves from a sinusoidally oscillating source consist of oscillating electric and magnetic fields. The electric and magnetic fields are always mutually perpendicular and oscillate in phase. The direction of propagation of an electromagnetic wave is perpendicular to both the $\vec{E}$ and $\vec{B}$ vectors. In other words, **EM waves are transverse.** Figure 4.8 shows the E and B components of an electromagnetic wave travelling along the x axis at a given instant.

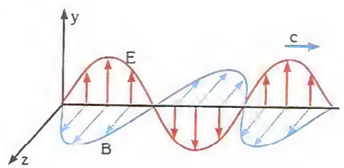


Figure 4.8 Electromagnetic wave

As can be seen from Figure 4.8, the $\vec{E}$ and $\vec{B}$ components reach maximum values and drop to zero simultaneously. **The electric and magnetic fields oscillate in phase.**

An electromagnetic wave should not be confused with a mechanical wave such as surface water waves. The EM wave does not have physical crests and troughs similar to the surface of water transmitting a wave. Rather, the sinusoidal curves in Figure 4.8 show the variation of the fields along the **x axis**. In other words, all the oscillation takes place along the x axis in Figure 4.8. For plane EM waves as shown in Figure 4.9, the electric and magnetic fields oscillate with the same amplitude and frequency at all points over a plane y-z at a fixed x coordinate. At a given instant, nothing changes along the y or z axes.

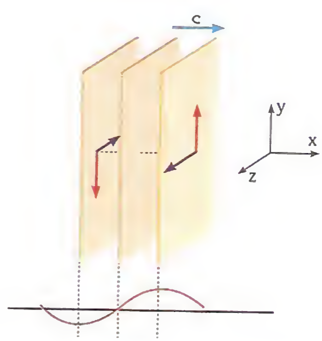


Figure 4.9 Plane wavefronts

A detailed calculation starting from Maxwell's equations for plane EM waves show that the magnitudes of electric and magnetic field components are always proportional to each other at a given position. The relationship is given by

$$E = cB$$

where E is expressed in volts/metre, B in Teslas and $c = 3\times10^{8}$ m/s.

For plane EM waves, the mutual orientations of the $\vec{E}$ and $\vec{B}$ fields and the velocity vector are always similar. Using the right hand rule by curling the fingers of the right hand in a sense of rotation from $\vec{E}$ to $\vec{B}$, the thumb points in the direction of the velocity vector, as shown in Figure 4.10.

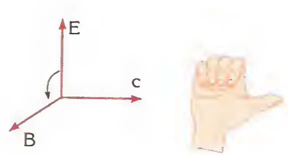


Figure 4.10 Mutual orientation of E, B and velocity vectors

The common characteristics of plane EM waves travelling in vacuum can be summarised as follows

- EM waves do not require a material medium
- EM waves consist of E and B fields oscillating in phase
- The speed of EM waves in a vacuum is independent of the frequency of the waves. All EM waves propagate with the same speed in a vacuum.
- EM waves are transverse. The $\vec{E}$ and $\vec{B}$ vectors and the velocity vectors are mutually perpendicular

EM waves reflect and refract at interfaces between dielectrics. EM waves also diffract and interfere, similar to mechanical waves. Reflection, refraction, interference and diffraction of light waves will be investigated in detail in the next chapter.

Frequency-Wavelength Relationship

The relationship

$$\text{speed} = \text{frequency} \times \text{wavelength}$$

for mechanical waves also holds for EM waves. Therefore, in vacuum,

$$c = f\lambda$$

for all EM waves. **Since the speed is constant, a higher frequency implies a shorter wavelength for all EM waves.**

4.3 THE ELECTROMAGNETIC (EM) SPECTRUM

The electromagnetic (EM) spectrum is an array of electromagnetic waves arranged in order of their frequencies.

EM waves share certain common properties as described above. However, different regions of the EM spectrum are labelled according to:

i. Frequency (or wavelength)
ii. Source or production method
iii. Effects and usage

Figure 4.11 below shows the full electromagnetic spectrum. The frequency range of EM waves is huge. It is possible to detect waves of frequency as small as 0.01 Hz. On the other hand gamma rays may have frequencies higher than 10^{23} Hz.

Notice that the boundaries between neighbouring regions in the spectrum are not definite. An electromagnetic wave of a given frequency can be called ultraviolet or X-ray depending on its method of production..

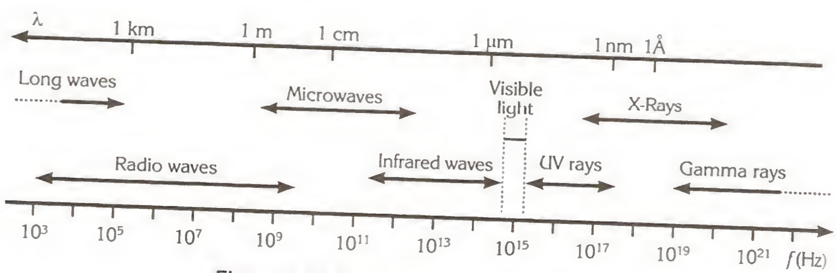


Figure 4.11 The electromagnetic spectrum

Long waves are produced by AC power transmission lines. The household AC current has a frequency of 50 Hz. Accordingly, EM waves of 50 Hz frequency are sent out from alternating current (AC) carrying cables.

Radio waves range between 0.5 cm and 30 km of wavelength. Radio waves are used in radio and TV communication.

Radio waves can be electronically generated by oscillating charges along an antenna. Transmission and detection of radio waves are examined in Section 4.5.

The radio wave region is divided into sub-regions as shown in Table 4.1 below.

Radio waves				
Long Wave	**Medium Wave**	**Short Wave**		
Low Frequency (LF)	Medium Frequency (MF)	High Frequency (HF)	Very High Frequency (VHF)	Ultra High Frequency (UHF)
10 km>λ >1 km	1 km>λ >100m	100 m >λ >10m	10 m >λ > 1m	1m >λ >0.1m
30 kHz<f<300 kHz	300 kHz<f<3MHz	3MHz<f<30MHz	30MHz<f<300MHz	300 MHz<f<3000 MHz
Long wave AM broadcasting	Medium wave AM broadcasting	Shortwave radios	Television and FM broadcasting	Television, wireless LAN, mobile phones.
Follows the Earth's curvature via interactions with the ground and ionosphere		Reflected from ionosphere	Line of sight	Line of sight

Table 4.1 Radio waves

Radio waves of wavelength longer than 100 m (medium and long wave) has an ability to follow the Earth's curvature for long distances via interactions with the ground and the ionosphere. This property becomes less apparent in shorter wavelengths. Thus, for long distance (out of eye contact) communications on the Earth, generally radio waves of wavelength longer than 100 m are used.

A layer of the ionosphere, occurring from about 90 to 150 kilometres above Earth makes long-distance communications possible by strongly reflecting radio waves in the range from one to thirty MHz (300 m >λ >10 m), as shown in Figure 4.12. High frequency (HF) waves can reach very distant points on the Earth, owing to multiple reflections from the ionosphere and Earth's surface.

Figure 4.12 Radio waves are reflected by the ionosphere (not to scale)

The ionosphere is transparent to radio waves with wavelengths shorter than 10 m, hence this region of radio waves is employed in short distance communications in the range of eye-contact, and in communication with satellites.

Microwaves are also called **radar waves**. Microwaves are electromagnetic waves in the range of wavelength between 10 cm and 1mm. The shortest wavelength (highest frequency) microwave sets the limit of electromagnetic waves that can be generated electronically. To produce waves of higher frequency, one would need electronic circuits of molecular size.

Radar waves are employed in object location. A radar station sends millions of short electromagnetic pulses per second in a directed beam. The waves reflecting from any obstacle (such as an airplane) are detected by receivers also located at the radar station. Assuming the interval between transmission and the detection of a signal is τ, the distance between the station and object is given by

$$d = \frac{c\tau}{2}$$

Infrared waves have a lower frequency (longer wavelength) than visible waves. The infrared range lies between the longest wavelength visible light (7.5×10^{-7}m) and the shortest wavelength radar waves (about 1 mm). The source of infrared radiation is the (thermal) motion of atoms and molecules in a substance and the motion of electrons in an atom. IR waves are emitted by all objects in proportion to their temperature.

Some animals can "see" infrared radiation. Thus, they can go hunting in the dark, detecting the infrared radiation emitted by warm-blooded animals.

Infrared radiation is absorbed by most objects. The energy absorbed from IR radiation becomes the thermal energy of the object. This is why infrared waves are also called "heat waves". We cannot see IR rays but we can feel the warmth caused by infrared waves from a fire or radiator.

Visible light can be seen by the human eye. The visible wavelength range lies between 4.3×10^{-7} m (violet) and 7.5×10^{-7}m (red). The eye retina is actually an electric device adjusted to be sensitive to the electric field oscillating in this range. The variation of wavelength in this range corresponds to different colours.

The source of light energy is the electrons moving between orbits in the atom. Light waves are mainly produced by very hot objects (i.e. filament lamps) and electric discharges through low density gases (i.e. neon lamps).

The visible section constitutes a tiny portion of the whole EM spectrum. The eye is the only organ in our body capable of processing a portion of the EM wave spectrum. The "ordinary" human eye converts light energy into electrical bits of information at an unbelievable rate. However, we are virtually blind to the remaining portion of the EM wave spectrum, although we live in a "sea" of EM waves from various sources. It might be interesting to try to imagine how the world around us would seem, if we had other organs capable of "seeing" other types of electromagnetic waves.

Ultraviolet waves have a higher frequency (shorter wavelength) than light waves. UV rays originate from energy changes of electrons in the atom.
Ultraviolet waves, x-ray waves and gamma rays can be harmful to living things. UV is used for disinfection, since it kills bacteria.

Tanning of the skin is caused by UV waves from the Sun, which is the main source of UV radiation. Most of the UV radiation from the Sun is filtered by the ozone layer of the atmosphere. Long term exposure to UV may cause skin cancer. Electric arcing that occurs during welding also produces UV radiation. Welders must therefore wear special goggles, as the UV rays are harmful to the human eye.

<table>
<tr><th></th><th>Radio</th><th>Radar Waves</th><th>Infrared Waves</th><th>Visible Waves</th><th>UV Rays</th><th>X-Rays</th><th>Gamma Rays</th></tr>
<tr><td>Wavelength</td><td>Km- 30 cm</td><td>30 cm- 1 mm</td><td>1 mm- 7×10^{-7}m</td><td>7×10^{-7} m- 4×10^{-7} m</td><td>4×10^{-7} m- 6×10^{-10}m</td><td>10^{-8} m- 10^{-10}m</td><td>10^{-10} m- 10^{-14} m</td></tr>
<tr><td rowspan="2">Source</td><td colspan="2" rowspan="2">Accelerate charges in antennas via electronic devices</td><td>Warm bodies, vibrating molecules</td><td>Very hot objects, discharge tubes</td><td>Radiated by the Sun, electric discharges</td><td rowspan="2">Fast electrons striking a metal target</td><td rowspan="2">Radioactive nuclei</td></tr>
<tr><td colspan="3">Electrons in atom</td></tr>
<tr><td rowspan="2">Usage</td><td rowspan="2">Communication</td><td rowspan="2">Radio detection, cooking (Also called microwaves)</td><td rowspan="2">Heats objects, physiotherapy, photography</td><td rowspan="2">Visible</td><td>Disinfects, causes sun-tans</td><td>Medical</td><td>Kills people treats cancer</td></tr>
<tr><td colspan="3">Harmful for life</td></tr>
</table>

Table 4.2 Some properties of electromagnetic waves

X-rays have wavelengths shorter than 10^{-8}m. X-rays originate from sudden deceleration of high energy electrons striking metal targets, or from energy changes of electrons in the atom. X-rays can penetrate living tissue and are used in medicine.

Gamma rays generally originate from the nucleus during nuclear reactions. Gamma rays differ from x-rays in their source.

4.4 THE ENERGY CARRIED BY EM WAVES

Intensity

The intensity of a wave is the energy carried by the wave through unit area perpendicular to the rays in unit time. Thus, intensity has the dimensions of

$$\frac{\text{Energy}}{\text{time} \cdot \text{area}} \quad \text{or} \quad \frac{\text{power}}{\text{area}}$$

The SI unit of intensity is watt/m^2.

The Intensity - Energy Density Relationship

Energy density is defined as the energy per unit volume. The defining formula is:

$$u = \frac{\text{Energy}}{\text{Volume}}$$

where "u" stands for energy density. The SI unit for energy density is J/m^3.

Consider the EM wave passing through an imaginary circular area "A", as shown in Figure 4.13. The direction of wave motion is perpendicular to the area. In the time interval Δt, the wave moves a distance of $c\Delta t$. Therefore, the energy flowing past the circular area in time Δt is contained in the cylindrical volume of $\Delta V = Ac\Delta t$. By definition, the energy (ΔW) contained in this volume is given by

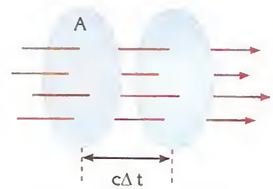


Figure 4.13 The volume of a cylinder is equal to its base x height

$$\Delta W = u\Delta V = ucA\Delta t$$

The intensity of energy flow equals

$$I = \frac{\Delta W}{A\Delta t} = \frac{ucA\Delta t}{A\Delta t}$$

$$I = uc$$

The intensity of EM waves is the product of energy density and wave speed.

The Intensity - Frequency Relationship

The intensity of sound waves, or the power transmitted by the waves along a spring is proportional to the amplitude squared, as was stated in Section 2.2. In general

$$\text{Intensity} \sim \text{Amplitude}^2$$

There is a similar relationship between amplitude and wave intensity also for EM waves. The intensity of EM waves is proportional to the electric field squared, or magnetic field squared.

$$I \sim E^2 \sim B^2$$

The (electric or magnetic field) amplitude of an EM wave originating from a harmonically oscillating charge is proportional to the acceleration of the charge

$$E \sim \text{acceleration}$$

In simple harmonic motion, acceleration is proportional to the square of the frequency of oscillations, as stated in Section 1.5. Therefore the amplitude of the electric field of an oscillating charge is proportional to the square of the oscillation frequency.

$$E \sim \omega^2$$

Consequently, the intensity of EM waves is proportional to the fourth power of frequency of the waves

$$\text{Intensity} \sim \omega^4$$

The Intensity - Distance Relationship

The intensity of EM waves from a source may depend on the direction of the waves, as is the case with radio transmitter antennas. Such a case is difficult to analyse.

To simplify the analysis, assume that an EM wave source emits radiation of equal intensity in all directions. Assume also that the intensity is measured at a large distance from the source, compared to its size. Such an ideal source is called a "point source" of radiation.

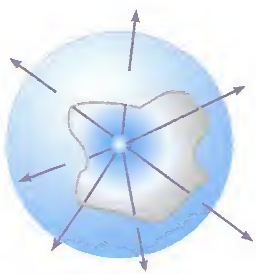


Figure 4.14 Power emitted by the source is distributed over a spherical surface

The radiation intensity from a point source has a simple dependence on distance. Since the radiation is spherically symmetrical, at a distance R away from the source, the total power emitted by the source is uniformly distributed over a sphere of surface area $4\pi R^2$, as shown in Figure 4.14. The intensity is given by

$$I = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi R^2}$$

$$I \sim \frac{1}{R^2}$$

Therefore the intensity of EM waves from a point source is inversely proportional to the square of the distance from the source.

4.5 THE TRANSMISSION AND DETECTION OF RADIO WAVES

Waves from an Antenna

The variable current in the LC circuit discussed in Chapter 3 is caused by the electric charges accelerating in the circuit. Thus the oscillating LC circuit is a source of electromagnetic waves. The sinusoidal oscillations in the LC circuit give rise to sinusoidal electromagnetic waves. The frequency of electromagnetic waves produced by oscillating charges in a circuit equals the frequency of charge oscillations in the circuit. The frequency of oscillations in an LC circuit is given by

$$\omega = \frac{1}{\sqrt{LC}}$$

Since the source frequency equals the wave frequency, the frequency of EM waves produced by an oscillating LC circuit is also given by the same formula.

The magnitude of the induced electric field depends on the rate of change of the magnetic field, whereas the magnitude of the induced magnetic field depends on

the rate of change of the electric field. Therefore to generate high energy (large magnitude) EM waves, high frequency sources are needed. A typical radio wave frequency lies between 100 - 1000 MHz. To produce such high frequency waves, the inductance and capacitance values of an LC circuit must be very small. Radio and radar waves can be produced by electric charges accelerating along a conductor called an "**antenna**". An antenna generally consists of a long conductor coupled to an oscillator as shown in Figure 4.15.

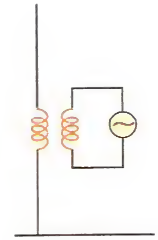


Figure 4.15 A Transmitting antenna

Any antenna can be modelled as an "open" LC circuit. Figure 4.16 demonstrates this fact. When charges oscillate harmonically along an antenna, there exists a current along the extended conductor, similar to the current through the inductor of an LC circuit. The two extremities of the antenna are sinusoidally charged and discharged, similar to the charging of two plates of a capacitor. The current along an antenna is not uniform as it was in the LC circuit. The maximum current occurs in the middle part of the conductor and the current drops to zero near the extremities.

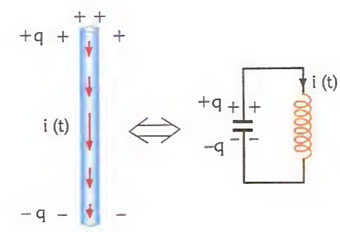


Figure 4.16 An antenna can be viewed as an LC circuit. The two ends act like the plates of a capacitor

Consider an antenna consisting of a long conductor coupled to an oscillator. Close to the antenna, the electric field changes in accordance with the charge accumulation at the extremities as shown in Figure 4.17 (a). Similarly, the magnetic field produced by the current along the conductor changes in step with the currents as shown in Figure 4.17 (b). The behaviour of an EM field near the antenna (called the "near field") is complicated. The dominant near field $\vec{E}$ and $\vec{B}$ components do not radiate energy.

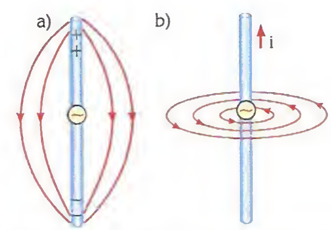


Figure 4.17 a) The electric field near an antenna produced by the charge accumulated near the extremities during an oscillation. b) The magnetic field near an antenna produced by the current along the conduc-

Away from the source, the EM wave is independent of the source. The electric field component of the EM wave oscillates parallel to the antenna whereas the oscillations of the magnetic field takes place in a plane perpendicular to the antenna, as shown in Figure 4.18.

The EM wave intensity takes its maximum value in directions perpendicular to the antenna. Along the line of the antenna the intensity drops to zero.

Detection of Radio Waves

Simplified versions of two types of receiving antennas for radio waves are shown in Figure 4.19 (a) and 4.20.

Figure 4.19 (b), shows the interaction of the electric field component of the EM wave with the electrons in the antenna. The electrons are forced to oscillate with the same frequency as the wave frequency.

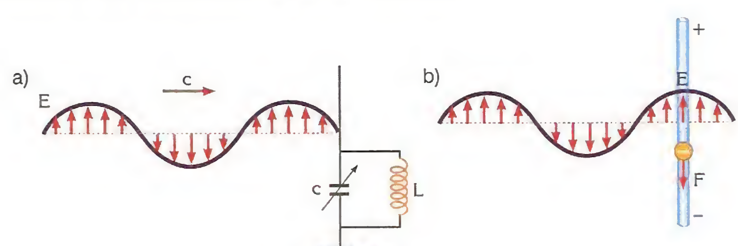


Figure 4.19 a) Receiving antenna for the detection of the electric field oscillations b)The electric field acts on the free charges in the antenna

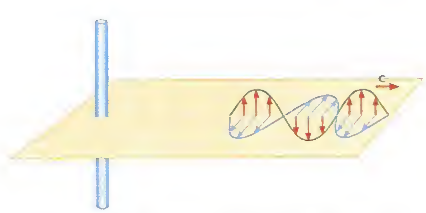


Figure 4.18 The respective orientations of E field, B field and antenna body for the EM wave at a distance of several wavelengths away from the antenna

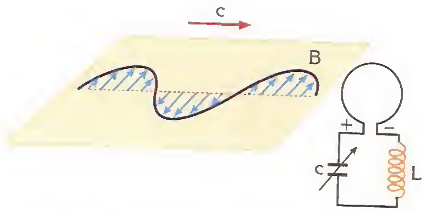


Figure 4.20 A Receiving "loop antenna", to detect the oscillating magnetic field

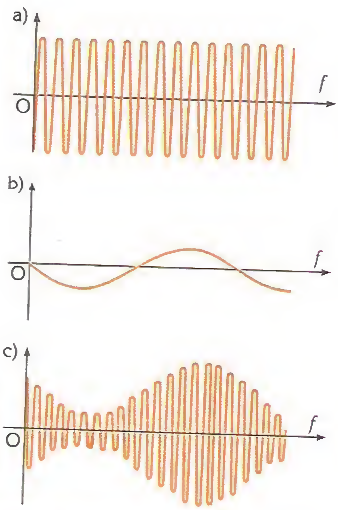


Figure 4.22 The amplitude modulation
a) The high frequency carrier signal
b) The low frequency signal containing the audio information
c) The modulated signal

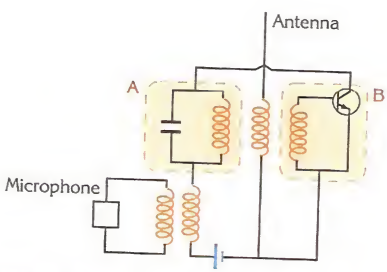


Figure 4.23 Circuit scheme of a simple transmitter

In Figure 4.20, the oscillating magnetic field induces current around the loop oscillating with the same frequency as the field.

In both cases, the antennas are coupled to LC circuits with a variable capacitor. EM waves from many different sources (i.e. radio stations with different frequencies) reach the antenna simultaneously. If an electromagnetic wave has a frequency equal to the natural frequency of the LC circuit, resonance occurs (For resonance see Section 1.9). The charges in the antenna circuit undergo forced oscillation with the same frequency as the wave frequency. The EM waves of other frequencies which do not match the natural frequency of the receiving antenna circuit are hence filtered out.

Transmission and detection circuits are examined in more detail in the next section.

4.6 AMPLITUDE MODULATION (Optional)

The Transmitter

Radio waves occur typically in the frequency range 100-1000 MHz. Long distance signal transmission is possible only with such high frequency waves which are intensely emitted from the antenna. The frequency of audible sound waves lies in the range 20-20 000 Hz. To transmit the low frequency audio signal, high frequency radio waves are used as "carrier waves".

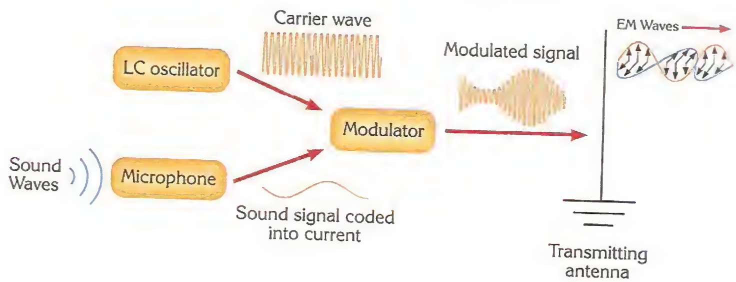


Figure 4.21 Amplitude modulation

Figure 4.21 schematically describes the process of modulation. A microphone converts the variations in air pressure into variations in electric potential. A circuit superposes the electric signal produced by the microphone and the high frequency alternating current produced by an oscillatory circuit. This process is called the "modulation" of the high frequency signal.

Figure 4.22 shows the high frequency carrier signal, the low frequency audio signal and the modulated signal. Since the amplitude of the carrier signal is varying, Figure 4.22 represents "amplitude modulation" or AM. The modulated current is then driven along an antenna, which issues EM waves of the same frequency as the current.

The circuit scheme of a simple AM transmitter is given in Figure 4.23. The component marked by "A" is the LC oscillator producing the high frequency carrier signal.

The component marked by "B" feeds the energy from the battery into the oscillator, without disturbing the oscillation frequency. Without energy supply, the oscillations die out quickly, owing to the resistance in the circuit and the energy radiated away in the form of EM waves.

The Receiver

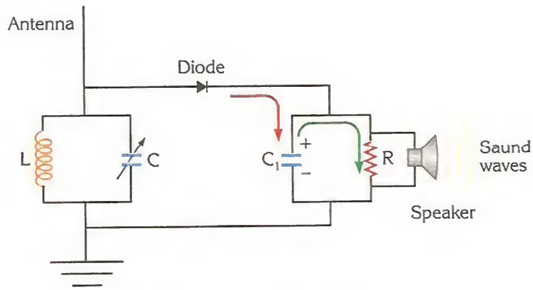


Figure 4.24 Circuit scheme of a simple receiver

Figure 4.24 shows the circuit scheme of a simple receiver. The LC circuit connected to the antenna "chooses" the radio wave having a frequency equal to its natural frequency, among many EM waves reaching the antenna. In the old style radios, turning the knob to choose a station actually changes the natural frequency of the LC circuit, by changing the value of the variable capacitor in the circuit. The chosen radio signal induces a potential difference across the antenna similar to the one shown in Figure 4.22 (c). The receiver "demodulates" the modulated signal. Demodulation means separating the low frequency wave impressed (or coded) with the audio signal from the high frequency carrier wave.

The circuit in Figure 4.24 demodulates the signal as follows:

- Assume the potential difference induced along the antenna is in the form shown in Figure 4.25 (a). The diode allows the current to flow only in one direction. When the potential difference across the antenna is negative, the diode blocks the flow of current. Thus the potential across the antenna induces a current through the diode as shown in Figure 4.25 (b).

- The capacitor indicated by C_1 in Figure 4.25 is charged by the current from the diode. The current charging the capacitor is shown by a red arrow in the circuit scheme. When the voltage across the antenna is reversed, the diode acts like an open circuit, but the current through the resistor still continues to flow in the same direction. This is because during the interval between the pulses, the capacitor discharges through the resistor, as indicated by the green arrow in Figure 4.24. The presence of resistance in the discharge process makes it much slower than charging. Thus, before the capacitor has enough time to discharge, the voltage across the antenna reverses and the current allowed by the diode fills the capacitor again.

The current through the resistor is as shown in Figure 4.25 (c).

- Therefore, the current flows through the resistor with the same frequency as the audio signal as shown in Figure 4.25 (d). Finally, a speaker converts the variation of current into sound waves.

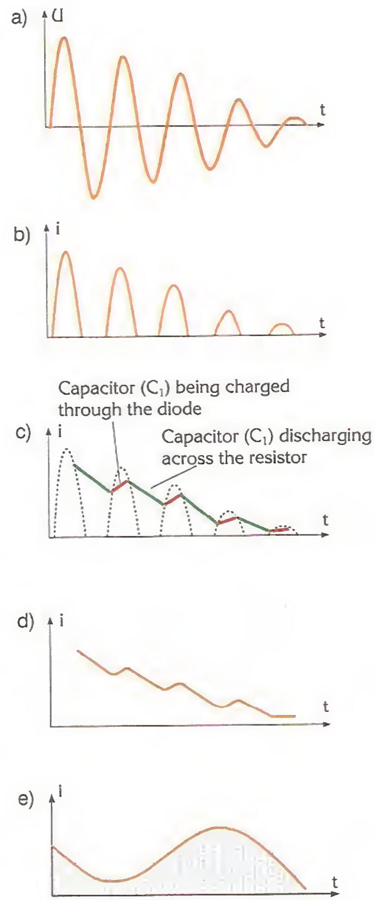


Figure 4.25 Demodulation process
a) Voltage across the antenna
b) Current through the diode
c) Current through the resistor
d, e) The signal coded with audio information is separated from the carrier signal

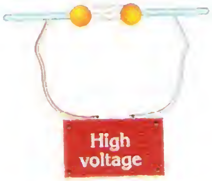


Figure 4.26 Hertz's transmitter signal

Figure 4.27 Hertz's receiver

Hertz's Studies

The existence of radio type electromagnetic waves theoretically proposed by Maxwell was first experimentally verified by Hertz in 1886, seven years after Maxwell's death.

To induce high frequency oscillations, Hertz used two long conductors welded to brightly polished brass spheres between which a small gap was left, as shown in Figure 4.26. The conductors are charged oppositely by means of an induction coil. When the voltage is high enough, the circuit is completed by a spark in the gap between the spheres; charge starts to oscillate in the circuit. Hertz was able to detect the EM waves by means of a receiver as shown in Figure 4.27. When two circuits are aligned parallel to each other, the oscillating electric field of EM waves causes the charge in the receiving circuit to oscillate. Sparks due to the voltage induced by the EM waves are observed in the receiver's gap.

Hertz measured the speed of EM waves, and showed that the radio range EM waves reflect, refract and interfere in a way similar to light waves.

Summary

A **changing magnetic field** generates an electric field.

A **changing electric field** generates a magnetic field.

Electromagnetic waves consist of changing electric and magnetic fields that reproduce each other and propagate in free space without requiring a medium.

Electromagnetic waves are produced by accelerating electric charges.

Properties of EM waves

EM waves propagate with the same speed in a vacuum; $c = 3\times10^8$ m/s

EM waves consist of E and B fields oscillating in phase.

EM waves do not require a material medium.

EM waves are transverse: $\vec{E}$, $\vec{B}$ and c vectors are mutually perpendicular.

EM waves reflect and refract at interfaces between dielectrics.

The speed-wavelength-frequency relationship

$$c=f\lambda$$

The intensity-energy density relationship

$$I=uc$$

The intensity-frequency relationship

$$I\sim\omega^4$$

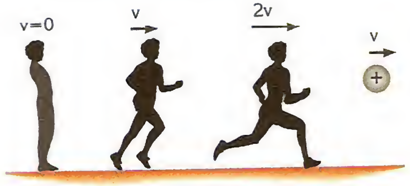
The intensity-distance from point source relationship

$$I=\frac{1}{R^2}$$

QUESTIONS AND PROBLEMS

4.1 Electric and Magnetic Fields

1.



Draw the magnetic field around the positively charged particle as observed by the three observers. The velocities indicated in the figure are with respect to the ground.

2. What is the condition for an electric charge to produce
   a) Electric field
   b) Magnetic field
   c) Electromagnetic waves

4.2 Electromagnetic Wave

3. Draw the relative orientations of the electric field, magnetic field and velocity vectors for an EM wave.

4. How did Maxwell infer from his theoretical calculations that light is a type of EM wave?

5. Under what circumstances does an electric charge emit EM waves?

6. What is the wavelength of 1.5 MHz radio waves?

7. The pendulum of an old style wall clock is positively charged. What is the wavelength of the EM waves as it swings?

8. A short-wave radio transmitter generates radio waves in the short wave range, $\lambda=10-100$ m. What is the frequency range of this transmitter?

4.3 The Electromagnetic Spectrum

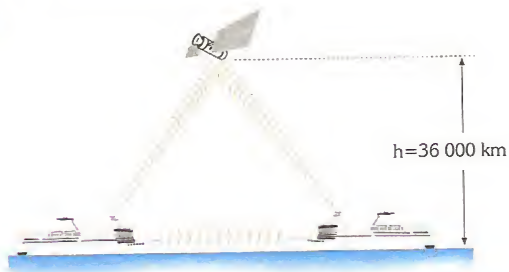
9. Arrange the given EM waves in order from low frequency to high frequency.
   a. Radar waves b. UV rays c. Gamma rays d. Radio waves e. Infrared rays f. X-Rays g. Visible waves

10. An airport radar system detects an unidentified flying object (UFO). What is the distance to the object if 100 μs passes between transmission and reception of radar signals?

11. The maximum value of the magnetic field component of an EM wave from a 100 MHz radio transmitter is $B_{max}=2\times10^{-11}$ T at a definite point in space.
    a) What is the maximum value of the electric component of the wave at the same point?
    b) How many wavelengths fit into the distance from the transmitter to the receiver 1 km away?

12. Radio waves having a wavelength of 2 m propagate in the +x direction.
    a) What is the frequency of the waves?
    b) What is the direction and magnitude of the electric field vector at a point when the magnetic field vector at the same point is in the +y direction and has an instantaneous value of $B=3\times10^{-7}$ T?

13.



The captains of two ships 170 m away from each other have a conversation by means of radio communication, while the crewmen just shout from ship to ship. The radio waves are reflected from a satellite at an altitude of 36 000 km from the surface of Earth.
   a) Which type of communication takes place faster?
   b) What is the time difference?

14. The minimum Jupiter-Earth distance is 588.5 million km. How long does it take a radar signal sent from an observatory to return to earth after reflecting from Jupiter?

15. A radar transmitter transmits 2000 short- duration radar signals (called pulses) per second. What is the range for determining the distance to an object using this transmitter? (Hint: Assume that to locate an object correctly, the pulse reflected from the object must have arrived at the receiver before the generation of the next pulse. Otherwise one cannot determine which of the pulses is received.)

16. A radar transmitter working in pulse mode produces 2500 discrete pulses in one second. The duration of each pulse is 1 μs. Find the maximum and minimum range of investigation for this transmitter.

17. An EM wave has $E(x,t)=(6\cdot 10^{-5}\text{V/m})\,\text{Sin}\left[10^{7}\cdot\left(t-\frac{x}{c}\right)\right]$ where c is the speed of light. Write the equation for the magnetic field wave. What is the frequency of the wave?

18. The amplitude of electric field oscillations in a sinusoidal EM wave from a 2 MHz transmitter is 100 V/m. Write the equation for the instantaneous value of electric-field E(t)=? Assume plane waves travelling in the +x direction.

19. The electric field component of an EM wave is given by $E=(200\,\text{V/m})\cdot\text{Sin}\left[2\pi(5\cdot 10^{-2}x-1.5\cdot 10^{7}t)\right]$. What are the frequency, wavelength and direction of propagation of the wave?

20. EM waves propagating in a vacuum enter a dielectric medium. Which parameters (v, f, λ) of the waves change?

21. Electromagnetic waves travel in a homogeneous medium with a speed of 2×10^{8} m/s. What is the wavelength of the waves in this medium, if the frequency in vacuum is 100 kHz?

22. Explain why inter-submarine communication is not possible via radio waves.

23. Why does one get more sunburn on a high mountain than at sea level?

4.4 The Energy Carried by EM Waves

24. What is the energy density of electromagnetic waves, the intensity of which is measured as 5×10^{-4} W/m^2?

25. What is the intensity of electromagnetic waves of an energy density of 10^{-10} J/m^3?

26. What is the intensity of radio waves at a distance 10 km away from a radio station of 50 kW power output? Assume that the source produces waves uniformly in all directions.

27. The radio wave intensity at a distance of 1 km from a radio station is 3×10^{-3} W/m^2. What is its intensity 10 km away in the same direction?

28. What is the approximate visible light intensity at a distance a 2 m from a 100-W light bulb? Assume that the bulb converts 10% of the total energy into visible light.

29. By what factor does the intensity of an EM wave change when the amplitude of the electric field decreases by half?

30. The maximum value of the electric field component of EM waves at a distance of 100 m away from a transmitter is 80 V/m. What is the amplitude of the electric field at a distance of 200 m from the transmitter? Assume that the radiation propagates in all directions uniformly.

31. The sound level 50 m away from an ideal (point) source of sound is 30 dB. Moving in the same direction, at what distance from the source does the level decrease to 10 dB?

32. An ideal speaker generates sound equally in all directions. The power of sound output is 100 W. What is the minimum distance from the speaker in order that the sound intensity is below the human pain threshold level of 120 dB?

33. Why, in practice, is an antenna used in place of an ordinary LC circuit (consisting of an inductor and a capacitor) to generate and transmit radio waves?

34. What is the period of oscillations in a transmitter antenna, generating EM waves at a frequency of 100 MHz?

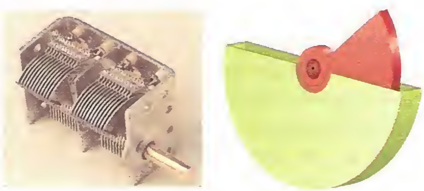
35. The instantaneous value of current in a transmitter circuit is given by $i(t)=0.2\text{Sin}\,(2\pi 10^6 t)$ in Amperes. What is the wavelength of radio waves generated by the antenna?

36. The capacitance and inductance values of a radio transmission circuit are 10 nF and 1 H, respectively. What is the frequency of radio waves generated by the circuit?

37. The maximum values of charge and current in an LC circuit are given as 10^{-6} C and 2 mA, respectively. What is the wavelength of the EM waves generated by the charge oscillating in the circuit?

38. The natural frequency of charge oscillations in a given LC circuit is 20 MHz. What is the frequency of the EM waves that will cause resonance?

39.



The tuning knob on an old style radio changes the effective area of the capacitor plates in the receiving LC circuit. Do we increase or decrease the effective plate area to tune in a radio station broadcasting at a longer wavelength?

40. A radio receiver circuit has an inductance of $L=0.5$ μH. To what value must the capacitance be adjusted to receive a station broadcasting at 100 MHz?

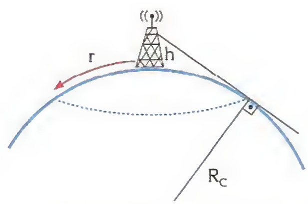
41. The tuning circuit of a radio receiver has an inductance of 10 μH. What is the capacitance of the circuit, if the radio receives a radio station broadcasting at a wavelength of 100 m?

42. A radio is tuned to receive a station broadcasting at a wavelength of $\lambda_1=100$ m. By what factor must we change the capacitance value of an LC circuit to receive another station broadcasting at $\lambda_2=400$ m? Assume that the inductance of the circuit remains constant.

43. An LC circuit in a radio receiver has a fixed self inductance value of 10 μH and a capacitance range between 1 nF and 10 μF. What are the longest and shortest wavelengths received by this radio?

44. A radio transmitter has a vertical antenna. How should the receiver antenna be oriented so that the reception is best?

45.



Show that the radius of "radio horizon" for an antenna is approximately given by $r=R_e\sqrt{2R_e h+h^2}$, where R_e is the radius of the Earth and h is the antenna height above the ground. The radius of radio horizon is the maximum distance "seen" by an antenna on a perfectly spherical Earth, as shown in the figure.

CHAPTER 5

Wave Optics

In geometric optics, light is assumed to be a stream of particles moving in a straight line. Geometric optics theory explains propagation, reflection and refraction of light, however, it does not make any assumptions about the nature of light. Diffraction, interference and polarisation of light cannot be explained by geometric optics. These phenomena can only be explained by the wave theory of light. For light to be accepted as a wave it must exhibit the properties which waves exhibit. In this chapter propagation, reflection, refraction, dispersion, interference, diffraction and polarisation of light waves will be discussed.

5.1 PROPERTIES OF LIGHT WAVES

According to the wave model, visible light is a form of electromagnetic wave. Thus, light waves are transverse similar to all the waves in the EM spectrum, as discussed in Chapter 4. The human eye is designed to detect EM waves in a limited range. This range is called the visible region. The speed of visible light is the same as the speed of all other electromagnetic waves in free space and is given by c = 300 000 km/s. The relationship between speed, frequency and is wavelength is:

$$c = \lambda f$$

The amplitude of light waves determines their intensity, which is perceived as the brightness of light. The frequency (or wavelength) of light waves is perceived as the colour of light by the human eye. For example red light has the lowest frequency (longest wavelength) which the human eye can detect and violet light has the highest frequency (shortest wavelength) which the human eye can detect (Figure 5.1).

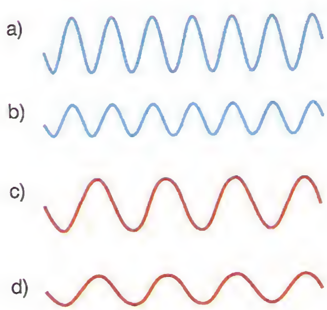


Figure 5.1
a) bright blue light
b) dim blue light
c) bright red light
d) dim red light

5.2 HUYGENS' PRINCIPLE

Huygens introduced a simple explanation of how wave theory works for both the laws of geometric optics and wave phenomena such as diffraction and interference. Huygens' principle states that **every point on a wavefront may be considered to be a source of secondary wavelets(small waves)**. All phenomena in this chapter can be explained by this principle. According to this principle the future shape and position of a wave is determined by points on its wave front; these points are the sources of secondary wavelets, the new wavefront is a surface which is a tangent to the surfaces of all secondary wavelets. A wavefront is perpendicular to the direction of propagation of a wave. Figure 5.2 shows how Huygens' principle is applied to the propagation of plane and circular waves. Consider the wavefronts AB as shown in Figure 5.2 a and b. All the points on this wavefront are sources of new wavelets, only a few wavelets are shown in the figure. After a short time new wavelets form a new wavefront, at A′B′.

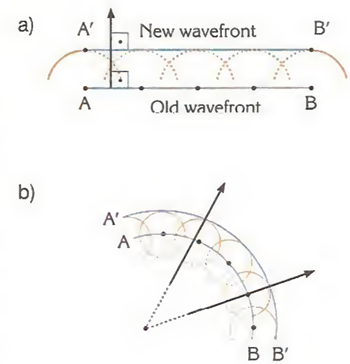


Figure 5.2 Huygens' Principle
a) propagation of straight waves
b) propagation of circular waves

Huygens explained reflection and refraction of light waves as follows. Consider the wavefronts incident on a boundary between two mediums as shown by the blue lines in Figure 5.3. All points on the wave front reach the boundary and form new wavelets on the boundary. The positions of two of these new wavelets a short time later is shown in the same figure. The wavelet produced at point B reaches the boundary at point B'. The red line in the figure shows the reflected wavefront. The right triangles AA'B' and ABB' are congruent triangles, since they have their shorter sides equal (AA' = BB') and a common hypotenuse (AB'), angles θ_1 and θ_2 are equal. This result coincides with, the law of reflection of light which states that $\theta_1=\theta_2$ as discussed in Section 2.6.

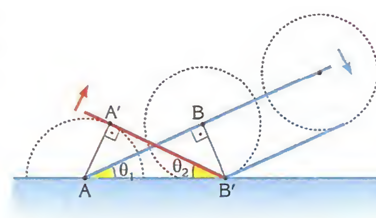


Figure 5.3 Derivation of the law of reflection by Huygens' Principle

The refraction of light waves also can be explained using Huygens' principle. Consider the same wave fronts discussed in Figure 5.3. The wavelets are transmitted into a new medium, as shown in Figure 5.4. Assume that the waves travel more slowly in the new medium. The wavelet produced at point A ,thus, expands less in the new medium. By drawing a tangent to the wavelet in the new medium at point B' the new form of the wavefront is obtained (Figure 5.4). To prove the law of refraction;

$$|AB'| = \frac{v_1 t}{\sin\theta_1} = \frac{v_2 t}{\sin\theta_2}$$

$$\frac{v_1}{v_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

since $v_1=\frac{c}{n_1}$ and $v_2=\frac{c}{n_2}$, where n is the absolute index of refraction, this equation can also be written in the form;

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} \text{ (Law of Refraction)}$$

Combining the above equation with that of Section 2.5

$$\frac{\text{Sin}\theta_1}{\text{Sin}\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} = n_{12}$$

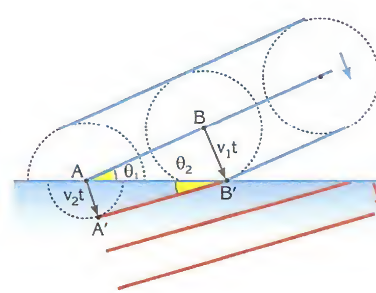


Figure 5.4 Derivation of the law of refraction by Huygens' Principle

5.3 DISPERSION

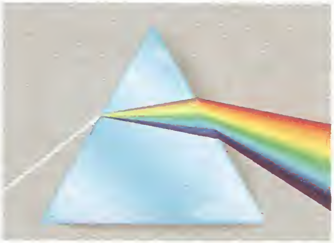


Figure 5.5 The dispersion of white light through a prism produces a spectrum

One of the most commonly observed optical phenomena is dispersion. The reason why light separates into different colours in a rainbow is due to dispersion. The visible spectrum contains many colours of different wavelengths. The combination of all these wavelengths produces white light. Likewise sunlight contains all the colours of the visible spectrum. On a rainy day sunlight refracts through the tiny droplets of water in the air and, if you are at the right position, you will observe a colourful rainbow. Newton also studied the spectrum; he separated sunlight into the colours of the rainbow after passing it through a prism, as shown in Figure 5.5. The separation of the sunlight into the colours of the rainbow can be explained by the wave theory of light. **Dispersion is the dependence of the index of refraction upon the wavelength or frequency of light.** Usually, the index of refraction is lower for longer wavelengths (or lower frequencies). For example, the refractive index of glass for red light is less than that for blue light, due to this, blue light refracts more than red light does.

5.4 INTERFERENCE AND COHERENCE

Section 2.7 discussed how two waves in the same medium can combine destructively or constructively at a point; this is called interference. Light from two sources must produce an interference pattern if light is, indeed, a wave. However, an interference pattern for two ordinary light bulbs is never observed. Ordinary light sources produce light via the thermal motion of atoms. Atoms in an ordinary light source gain energy by thermal collisions and release this energy as light in a very short time interval of the order of 10^{-8} seconds. There are many atoms in an ordinary light source which radiate in this way, so there is not a constant phase relationship between the light waves produced by atoms in this way. Thus, an interference pattern is unobservable. Sources which have a constant phase relationship between them can produce an observable interference pattern. Such sources are called **coherent** sources. For example, two loudspeakers connected to the same amplifier are coherent sources.

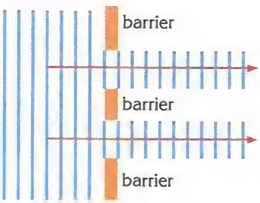


Figure 5.6 Waves passing through wide slits do not spread out enough to produce interference.

To produce an interference pattern of light a method which was first developed by Thomas Young is used. Two coherent light sources are generated by passing light from a single distant light source through two slits as shown in Figure 5.7. A distant light source must be used, since when the light source is close, wave fronts reaching the slit are still of a spherical shape. This disturbs the coherence of waves on the same plane. If the source is far enough, wavefronts reaching the slits are nearly plane and hence the coherency is not disturbed. However, in order that the light waves

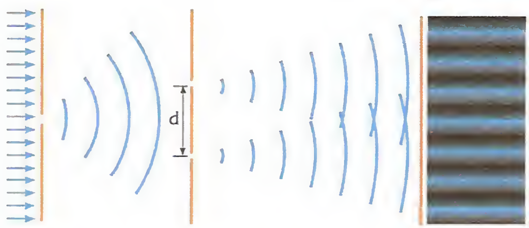


Figure 5.7 Young's double-slit experimental setup.

form an interference pattern, light from two sources must overlap at several points. If the slits are wide, light waves from the two slits do not interfere (Figure 5.6). Light waves passing through the slits must spread out in order to coincide at several points in space. Section 2.7 discussed how all mechanical waves spread out when they pass through an aperture, if the size of the aperture is comparable to the wavelength of the waves. Therefore in order for light waves to spread out, slits used must be narrow enough. This experiment is known as Young's double slit experiment.

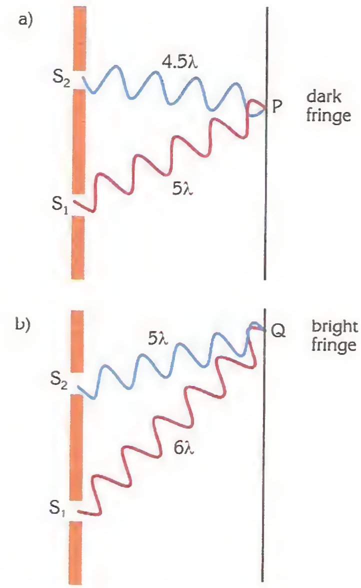


Figure 5.8 a) The formation of a dark fringe b) The formation of a bright fringe

Young's Double - Slit Experiment

The experimental set-up is shown in Figure 5.7. Monochromatic light passes through a narrow slit S. The waves spreading out from this slit reach a screen containing two narrow parallel slits, S_1 and S_2. Light waves emerging from the slits are in-phase, since the same wavefront reaches the slits at the same time. After passing through the slits the light waves fall onto a viewing screen which is a large distance away compared to the distance between the slits. The pattern formed on the screen is as shown in figure 5.7. The equally separated dark and bright bands on the screen are called **fringes**. Note that the central fringe is a bright fringe. These fringes are formed as follows:

Light waves from the slits reaching a point on the screen interfere constructively if they are in phase. The constructive interference forms a bright band on the screen which is also called a **maximum**.

Light waves from the slits reaching a point on the screen interfere destructively if they are completely out of phase. The destructive interference forms a dark band on the screen, which is also called a **minimum**. Figure 5.8 indicates schematically how a minimum and a maximum form on the screen.

In Figure 5.8-a waves from the slits reach point P. Since waves from slit S_1 travel a distance $\lambda/2$ further, waves from the two slits interfere destructively. Thus, a dark fringe is produced.

In Figure 5.8-b, waves from the slits reach point Q. This time since waves from slit S_1 travel a distance λ further, waves from the two slits interfere constructively. Thus, a bright fringe is produced at point Q.

The waves from two slits are in phase when they are first produced. The phase difference between the waves at a point occurs because the waves travel different distances from the sources to that point. **Path difference** to a point is defined as the difference of path length from the sources to that point. In Figure 5.9, the path difference to point P from the sources is

$$\text{path difference} = |r_1 - r_2|$$

In Section 2.7 the condition for minima and maxima for two source interference pattern was derived as follows

for minima	for maxima
$\lvert r_1 - r_2 \rvert = 0.5\lambda$	$\lvert r_1 - r_2 \rvert = 0$
$\lvert r_1 - r_2 \rvert = 1.5\lambda$	$\lvert r_1 - r_2 \rvert = 1\lambda$
$\lvert r_1 - r_2 \rvert = 2.5\lambda$	$\lvert r_1 - r_2 \rvert = 2\lambda$

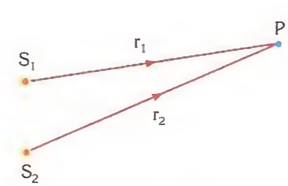


Figure 5.9 The path difference to point P is given by $|r_1 - r_2|$

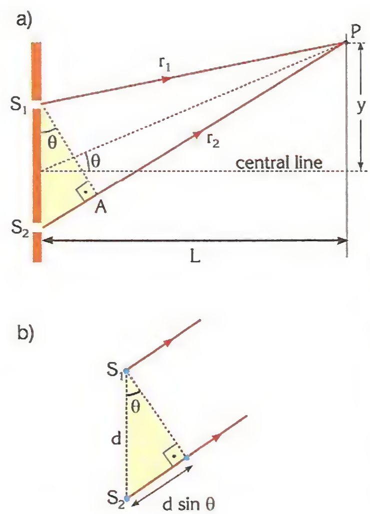


Figure 5.10 Ray diagrams for a double-slit experiment

So the condition for minima and maxima are as follows:

$$\text{for minima: } |r_1 - r_2| = (k - \frac{1}{2})\lambda \text{ , where } k = 1, 2, 3, \ldots$$

$$\text{for maxima: } |r_1 - r_2| = k\lambda, \qquad \text{where } k = 0, 1, 2, 3, \ldots$$

The integer "k" in the formulas shows the **order** of a fringe. Note that k=0 refers to the central bright fringe for which the phase difference is zero.

The double-slit experiment can be explained quantitatively with the aid of Figure 5.10. In the figure, monochromatic light is passed through two slits S_1 and S_2, which are separated by a distance "d", it then reaches a screen which is at a distance of L from the slits. Since distance L is very large compared to the slit separation d, light rays from slits S_1 and S_2 can be taken to be parallel (Figure 5.10-b). This approximation is called the Fraunhofer approximation. To make the light rays parallel a converging lens can also be placed in front of the slits.

In Figure 5.10-a the path difference is given by $|r_1 - r_2| = |S_1P - S_2P|$. Since $|S_1P| = |AP|$ the difference between the path lengths is $|S_2A|$.

$$|r_1 - r_2| = |S_2A|$$

$$|r_1 - r_2| = d\sin\theta$$

Since θ is very small, $\sin\theta \approx \tan\theta = \frac{y}{L}$, where y is the distance from the central line to point P, as shown in Figure 5.10-a.

Therefore the path difference of a point at a distance y from the central line is,

$$|r_1 - r_2| = d\sin\theta \cong d\frac{y}{L}$$

The condition for two source interference maxima and minima can be summarised as follows:

for minima	for maxima
$|r_1 - r_2| - \left(k - \frac{1}{2}\right)\lambda$ $d\sin\theta = \left(k - \frac{1}{2}\right)\lambda$ $d\frac{y}{L} = \left(k - \frac{1}{2}\right)\lambda$ $y = \left(k - \frac{1}{2}\right)\frac{L\lambda}{d}$	$|r_1 - r_2| = k\lambda$ $d\sin\theta = k\lambda$ $d\frac{y}{L} = k\lambda$ $y = k\frac{L\lambda}{d}$
$k = 1, 2, 3, \ldots$	$k = \underbrace{0}_{\text{central bright}}, 1, 2, 3, \ldots$

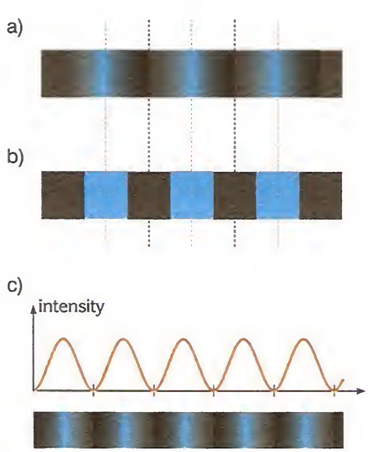


Figure 5.11 a) The appearance of the actual interference pattern. b) Schematic representation c) Intensity distribution of an interference pattern.

The Intensity of an Interference Pattern

An interference pattern consists of equally separated bright and dark fringes. However, this occurs only when θ is small and the distance from the slits to the screen, L, is very large compared to the slit separation. If the slit width is very small (comparable to wavelength) the intensities of all bright fringes are equal. The intensity distribution of an interference pattern is shown in Figure 5.11. As indicated in the graph, the intensity is a maximum at the centre of each bright fringe and a minimum at the centre of each dark fringe. Thus, when locating the position of maxima and minima, the distances from their centres must be found. A change in intensity of an interference pattern is caused by the phase difference between the light waves arriving from the slits. As discussed in Section 4.4, the intensity of a light wave is directly proportional to the square of the wave amplitude. Thus, the intensity at a point on an interference pattern is directly proportional to the square of the resultant wave amplitude at that point.

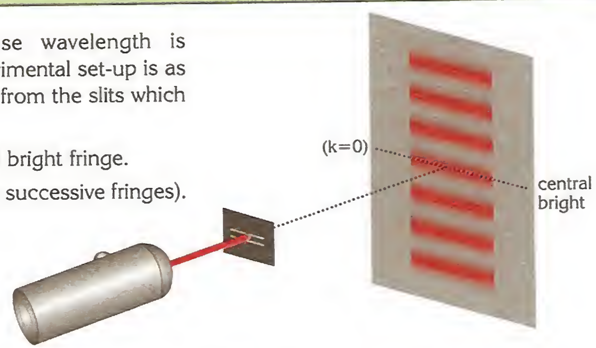
Example 5.1

Fringe patterns

In a double-slit experiment, a laser pointer (whose wavelength is $\lambda = 650$ nm) is used as a coherent light source. The experimental set-up is as shown in the figure. The screen is placed one metre away from the slits which are separated by a distance d = 0.1 mm.

a) Find the position of the 2nd dark fringe from the central bright fringe.

b) Find the fringe separation, Δy (the distance between two successive fringes).



Solution

a) Since the position of the 2nd dark fringe is required, k=2.

The wavelength is λ=650 nm=650×10^{-9} m, L=1 m and d=0,1 mm=$0,1\times10^{-3}$ m=10^{-4} m

$$y = (k - \frac{1}{2})\frac{L\lambda}{d}$$

$$y_{2(dark)} = (2 - \frac{1}{2})\frac{(1\text{m})(650\times10^{-9}\text{m})}{(10^{-4}\text{m})}$$

$$y_{2(dark)} = 975\times10^{-5}\text{ m}$$

$$y_{2(dark)} = 9.75 \text{ mm}$$

In the figure aside, the distance y_2 is then 9.75 mm.

b) The distance between the 1st and 2nd fringes or the distance between the 2nd and 3rd fringes or the distance between the 3rd and 4th fringes is called the fringe separation. The distance between two successive fringes is difference between the their respective distances from the central bright fringe. That is:

$$y_2 - y_1 \text{ or } y_3 - y_2 \text{ or } y_4 - y_3$$

Using one of these

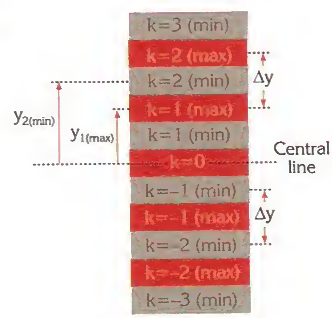
$$\Delta y = y_2 - y_1 = 2\frac{L\lambda}{d} - 1\frac{L\lambda}{d}$$

$$\Delta y = \frac{L\lambda}{d}$$

$$\Delta y = \frac{(1\text{m})(650\times10^{-9}\text{m})}{(10^{-4}\text{m})}$$

$$\Delta y = 650\times10^{-5}\text{ m}$$

$$\Delta y = 6.5\text{mm}$$



Example 5.2

a) What should be done to increase the fringe separation in a double-slit experiment?

b) What type of pattern is obtained when white light is used as a source in the double-slit experiment?

Solution

a) Fringe separation depends on L, λ and d. In example 5.1 it was found that

$$\Delta y = \frac{L\lambda}{d}$$

According to this equation it is obvious that increasing L, increasing λ or decreasing d, increases the fringe separation. Therefore to produce wider fringes on the screen, light of longer wavelength must be used or the slit separation must be decreased.

b) White light contains different colours. So using white light in a double-slit experiment produces different interference patterns which overlap on the same screen. Consider the interference patterns of blue and red light falling on the same screen. The separate patterns produced by red and blue light and their composite pattern; together are shown in Figure 5.12-a. Thus, all the colours between red and blue are included, white light produces a colourful pattern on the screen, as shown in Figure 5.12-b.

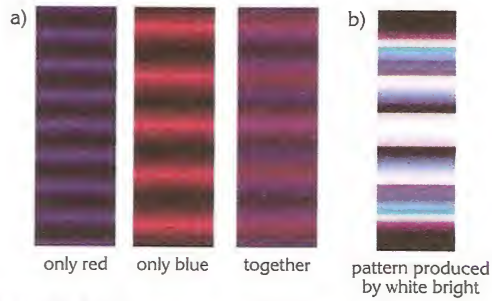


Figure 5.12 a) Interference pattern of two colours; red and blue b) Interference pattern produced by white light.

5.5 DIFFRACTION GRATINGS

A diffraction grating consists of many parallel slits of equal width. The slits are spaced equally. When monochromatic light passes through a diffraction grating, thin bright lines occur on a viewing screen. These equally spaced thin lines are called interference maximums.

What happens when light passes through a diffraction grating? Figure 5.13 describes the principles. Monochromatic coherent light is incident on the diffraction grating, assume that the slits of the grating are very narrow. Since the light from narrow slits spreads out widely, light from all of the slits interfere. It is also assumed that light rays from all slits are parallel, since the distance from the grating to the screen is very large compared to the slit separation.

A bright fringe forms in the middle of the interference pattern. Constructive interference also occurs at a point on the screen where the path difference from two adjacent slits is an integer multiple of λ (as was described in double-slit interference). In other words, the path difference to that point from any two slits is an integer multiple of λ. So the condition for maxima is:

$$k\lambda = d\sin\theta \quad (k=0,\ 1,\ 2,\ 3,\ \ldots)$$

If the path difference from two adjacent slits to a point on the screen is different from $k\lambda$ destructive interference occurs.

The intensity distribution of an interference pattern is as shown in Figure 5.14.

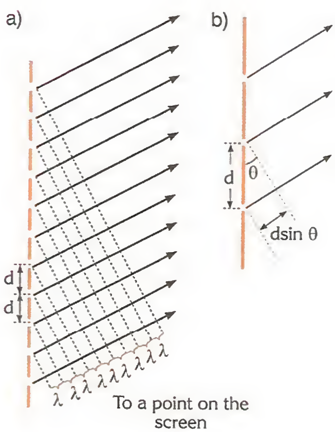


Figure 5.13 a) Light waves leaving the diffraction grating. The path difference to the screen, between adjacent slits, determines the pattern. This figure describes the first maximum. b) The path difference between two adjacent slits is d sin θ.

If white light passes through a diffraction grating each colour component has a maximum at a different location on the interference pattern. Thus, a colourful pattern is observed. This is why many colours are observed on a compact disc when illuminated with white light (Figure 5.15). Data on a compact disc is stored by means of pits (small notches). These pits are closely spaced so that they act like a diffraction grating.

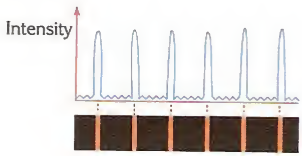


Figure 5.14 The intensity distribution of an interference pattern caused by a diffraction grating.

Figure 5.15 The colourful pattern of a CD is due to the interference of light from its pits.

Example 5.3 — Diffraction grating

A diffraction grating has 800 slits in 1 cm.

a) Find the slit spacing.

b) Find the wavelength of the light if the 3rd maximum is observed at an angle of 9°. (Sin 9°=0.156)

Solution

a) If 1 cm contains 800 slits then the separation of two successive slits is $\frac{1}{800}$ of 1 cm. That is

$$d = \frac{1}{800}\,\text{cm} = 1.25\times10^{-5}\ \text{m}$$

b) Since the angular position of the 3rd maximum is given, k=3.

$$k\lambda = d\sin\theta = d\sin 9^\circ$$

$$3\lambda = (1.85\times10^{-5}\,\text{m})(0.156) = 1.95\times10^{-6}\,\text{m}$$

$$3\lambda = 1950\,\text{nm}$$

$$\lambda = \frac{1950\,\text{nm}}{3}$$

$$\lambda = 650\,\text{nm}$$

5.6 THIN FILMS, NEWTON'S RINGS

Interference is not limited to double-slits or the diffraction grating. There are also many other examples of interference in nature; such as the colourful patterns on a soap bubble, thin oil layers on water surfaces, the vivid colours on the feathers of a peacock, or a butterfly's wings. Since a thin film has two surfaces, when a light wave reflects separately from these two surfaces, the reflected waves will have a phase difference between them. The reason why light forms an interference pattern on a thin film (very thin layer of transparent medium) is the phase difference between the reflected light waves. The interference pattern on the surface of a soap bubble is shown in Figure 5.16.

Figure 5.16 The colourful pattern on the surface of a soap bubble is due to thin film interference.

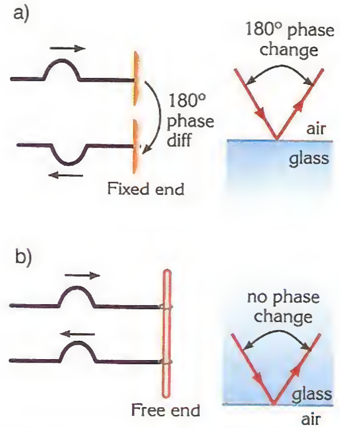


Figure 5.17 a) A light ray travelling in an air medium and incident upon a glass medium is reflected with a phase difference of 180°. b) Since glass has a higher refractive index than air light travelling in the glass medium, and incident on an air medium is reflected with no phase change.

The reflection of light waves from the surface of a thin film is similar to the reflection of waves on springs, which was discussed in Section 2.6. Assume light waves travelling in a transparent medium are reflected from a second medium. **If the first medium has a lower index of refraction, the reflected wave is subject to a phase difference of 180° and if the first medium has a higher index of refraction, the reflected wave is not subject to a phase change** (Figure 5.17). The principles of how light waves cancel or reinforce in a thin film will now be discussed..

Consider a glass coated with a thin film of thickness d and refractive index n (Figure 5.18). For simplification, assume that incident light rays are nearly perpendicular to the surface of the film. If there are two reflected rays; one reflected from the surface of the film and the other reflected from the surface of the glass, since both rays are reflected from a medium of higher refractive index, they both undergo a 180° phase difference compared to the incident light ray. However, there is another factor which causes a phase difference: path difference. Ray 2 travels further than ray 1 by a distance of approximately 2d.

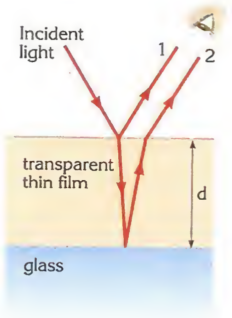


Figure 5.18 The path difference between the first and second reflected waves is 2d, because ray 2 takes a longer path than ray 1 by 2d.

The rays are still in phase when they leave the film if the path difference is an integer multiple of λ. Since **a path difference of λ corresponds to a phase difference of 2π**, constructive interference results. Therefore:

$$\text{for constructive interference: } 2d = \lambda, 2\lambda, 3\lambda, \ldots$$

$$\text{for destructive interference: } 2d = \frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, \ldots$$

Notice that, in the above equations the wavelength of light in the thin film is given by;

$$\lambda_{film} = \frac{\lambda_{air}}{n_{film}}$$

The above equations are interchanged for a soap film in air. Since the ray reflected from the lower surface is not subject to a phase difference, the reflected rays will have an additional phase difference of π. Therefore the condition for constructive interference for a thin film in air is the condition for destructive interference for a thin film coating.

Thin films are of a great importance to manufacturers of optical instruments, because it is better for an optical instrument to collect more light. For this reason lenses in optical instruments are coated with a thin film which gives destructive interference for a wavelength near the middle of the visible spectrum..

One can also produce an interference pattern by using a layer of air as a thin film. Newton's rings is an example of this kind of interference. Newton's rings is the inter-

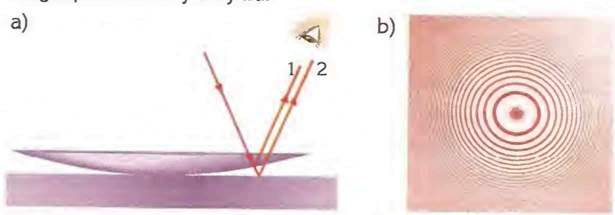


Figure 5.19 Newton's rings a) Side view b) Top view

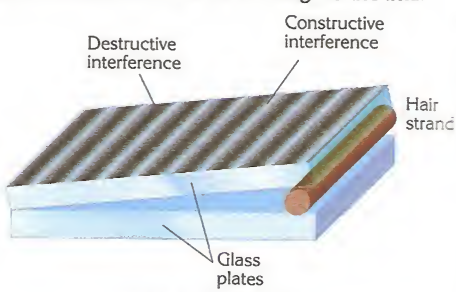


Figure 5.20 Air wedge

ference pattern formed by a thin layer of air between a flat glass plate and a lens as shown in Figure 5.19. The convex surface of the lens is in contact with the glass plate. As seen from the figure, the gap between the lens and the glass plate increases on moving away from the centre towards the sides. The width of this gap at a given location determines whether the interference is constructive or destructive. The interference pattern consists of many concentric dark and bright rings. Newton's rings is used as a method for testing optical lenses. Interference from a thin layer of air is also used to determine the diameter of very small objects. When a very small object is placed at the edge of two glass slides they produce a wedge shaped air gap that decreases with distance away from the object. For example, in order to find the thickness of a stand of hair, it is placed as shown in Figure 5.20, between two glass plates. When these plates are illuminated by a monochromatic light source from the top, bright and dark bands are observed. The thickness of the hair strand is determined by the number of these bands.

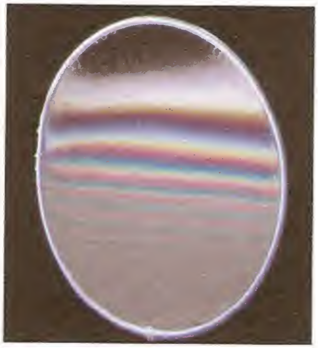


Figure 5.21 A soap film is illuminated by a white light source.

Example 5.4

A coloured soap film

Why is a soap film coloured under white light? See Figure 5.21

Solution

Due to gravity a soap bubble is thicker at the bottom, compared to the top. The varying thickness over the soap film leads to constructive interference at certain wavelengths contained in white light. Thus, a given thickness causes constructive interference for a given wavelength. As the soap film gets thicker, constructive interference is firstly observed for violet light and then for other colours, in spectral order. Since the top of the film is so thin, even the shortest wavelength cannot produce constructive interference.

5.7 SINGLE - SLIT DIFFRACTION

In section 2.7 diffraction was defined as the spreading of waves on passing through a narrow aperture or bending of waves around an obstacle. There are two main types of diffraction for light waves: Fraunhofer diffraction and Fresnel diffraction. When parallel light rays reach the screen from the slit, Fraunhofer diffraction results. To obtain this type of diffraction a lens is placed in front of the slit, and the screen is placed a large distance away. Fresnel diffraction results occurs for non-parallel light rays on a screen close to the slit, thus the wavefronts are still spherical. In this section Fraunhofer diffraction will be studied, since plane wavefronts are easier to analyse.

Suppose monochromatic light passes through a single slit of width "a" and a diffraction pattern on a viewing screen at a large distance "L" away from the slit as in Figure 5.22 is obtained.

In the middle of the pattern there is a wide bright fringe which is called the central bright fringe. Notice that the central bright fringe is wider than the width of the slit. On both sides of the central bright fringe, bright fringes of progressively lower intensity are produced. The intensity of the bright fringes decreases with increasing distance from the central bright fringe. However, most of the light falls on the central bright fringe. Therefore, by neglecting the other fringes, we can say that light spreads as wide as the central bright fringe.

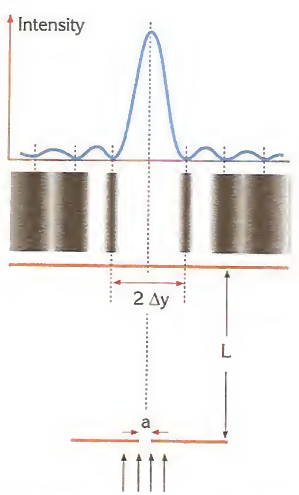


Figure 5.22 A single slit diffraction pattern

How do the bright and dark fringes form? Waves from two or more sources must interfere constructively or destructively to produce a pattern like the pattern produced in diffraction. To explain the formation of the interference pattern around the central bright fringe, recall Huygens' principle. Assume that each point on the wave front reaching the slit is a source of secondary waves. The superposition of all these secondary waves forms an interference pattern on the screen. In order to locate the minima and maxima around the central bright fringe, several points on the wavefront at the slit are taken which represent sources of in-phase secondary wavelets (Figure 5.23).

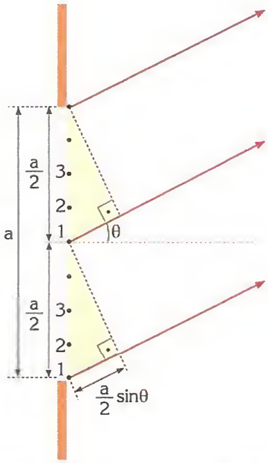


Figure 5.23 Locating the first dark in single slit diffraction

If the slit is divided into two parts, as in Figure 5.23, several pairs of points are obtained having a path difference $\frac{a}{2}\sin\theta$. between them. For example; the points numbered 1 in the top and bottom part of the slit have a path difference of $\frac{a}{2}\sin\theta$ between them. The points numbered 2 and the points numbered 3 also have this same path difference between them. If this path difference is equal to half a wavelength, waves from the two parts of the slit interfere destructively which results in a minimum (dark fringe):

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

$$a\sin\theta = \lambda \text{ (first dark)}$$

If the slit is divided into four portions the screen is dark when

$$\frac{a}{4}\sin\theta = \frac{\lambda}{2}$$

$$a\sin\theta = 2\lambda \text{ (second dark)}$$

If the slit is divided into six portions the screen is dark when

$$\frac{a}{6}\sin\theta = \frac{\lambda}{2}$$

$$a\sin\theta = 3\lambda \text{ (third dark)}$$

Therefore the general condition for minima (dark fringes) is:

$$a\sin\theta = k\lambda$$

since q is very small, $\sin\theta \approx \tan\theta = \frac{y}{L}$

$$a\frac{y}{L} = k\lambda$$

$$y_k = k\frac{L\lambda}{a} \quad (k=1, 2, 3, ...)$$

Where y_k is the distance from the k^{th} fringe to the central bright fringe.

Locating the first minima is important because the central bright fringe extends up until the first minimum on both sides of it. Therefore, the width of the central bright fringe is $2y_1$, where y_1 is the distance from the central line to the first minimum.

According to the equation above for the first minimum (k=1):

$$\sin\theta = \frac{\lambda}{a}$$

This result shows that the amount of diffraction depends on the ratio $\frac{\lambda}{a}$. One can find the amount of diffraction simply by finding 2θ. To produce clearer diffraction patterns, slit width "a" must be as small as possible. On the contrary, to reduce diffraction effects in optical instruments, like telescopes and microscopes, the ratio $\frac{\lambda}{a}$ must be as small as possible.

To locate the maxima (bright fringes) around the central bright fringe, the slit is divided into an odd number of portions. If the slit is divided into three portions, as shown in Figure 5.24, the path difference between corresponding points in two adjacent portions of the slit will be $\frac{a}{3}\sin\theta$. Thus, light waves from two portions of the slit cancel each other if their path difference equals $\frac{\lambda}{2}$, however, light from the third portion of the slit will still illuminate the screen. Thus, the screen is bright, if,

$$\frac{a}{3}\sin\theta = \frac{\lambda}{2}$$

$$a\sin\theta = 3\frac{\lambda}{2} \text{(first bright fringe)}$$

If the slit is divided into five portions:

$$\frac{a}{5}\sin\theta = \frac{\lambda}{2}$$

$$a\sin\theta = 5\frac{\lambda}{2} \text{(second bright fringe)}$$

If the slit is divided into seven portions:

$$\frac{a}{7}\sin\theta = \frac{\lambda}{2}$$

$$a\sin\theta = 7\frac{\lambda}{2} \text{(third bright fringe)}$$

So the general condition for maxima (bright fringes) is:

$$a\sin\theta = (k + \frac{1}{2})\lambda$$

$$a\frac{y}{L} = (k + \frac{1}{2})\lambda$$

$$y_k = (k + \frac{1}{2})\frac{L\lambda}{a} \quad (k=1, 2, 3, ...)$$

Where y_m is the distance from the m^{th} bright fringe to the central bright fringe.

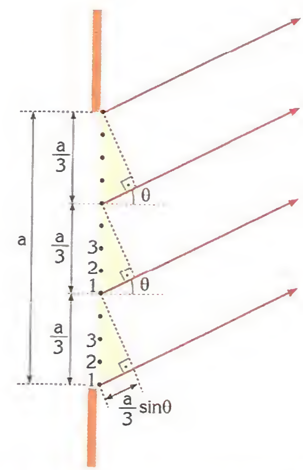


Figure 5.24 Locating the first bright in single slit diffraction

Example 5.5 Angular size of diffraction

A monochromatic light beam of wavelength 435 nm is passed through a 5 μm wide slit. How much does it spread?

Solution

Since most of the light falls on the central bright fringe in a single slit experiment, light is said to be spreading as much as its central bright fringe on the diffraction pattern. To find the spread of the central bright maximum the separation between the 1st dark fringes on both sides of the diffraction pattern must be found.

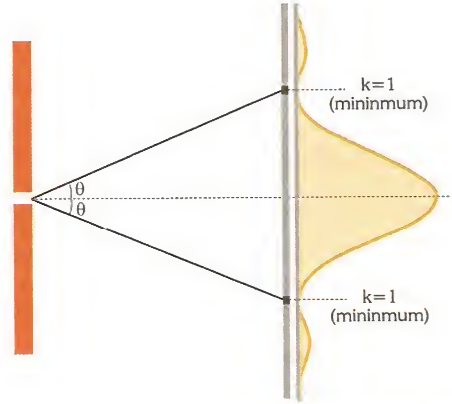
$$a\sin\theta = \lambda \text{ (first dark fringe)}$$

$$\sin\theta = \frac{\lambda}{a}$$

$$\sin\theta = \frac{(435\times10^{-9}\,\text{m})}{(5\times10^{-6}\,\text{m})}$$

$$\sin\theta = 87\times10^{-9+6} = 0.087$$

$$\theta = 5^{\circ}$$



This is the angular position of the first dark fringe on one side of the central maximum. The separation between the first dark fringes on both sides is 2θ. Therefore the width of the central fringe is 10°

$$2\theta = 10^{\circ}$$

Example 5.6 Fringe separation

A monochromatic light source of wavelength 600 nm passes through a 0.05 mm wide slit. Find the fringe separation and the width of the central bright fringe on a screen placed 2 m away from the slit.

Solution

Recall from example 5.1 that the distance between two successive fringes is the fringe separation. The position of the first and second dark fringes must be found in order to calculate the fringe separation.

$$\Delta y = y_2 - y_1 \qquad \text{(For minima, } y_m = k\frac{L\lambda}{a}\text{)}$$

$$\Delta y = 2\frac{L\lambda}{a} - 1\frac{L\lambda}{a}$$

$\Delta y = \frac{L\lambda}{a}$ Note that $\frac{L\lambda}{a}$ is the position of the first minimum.

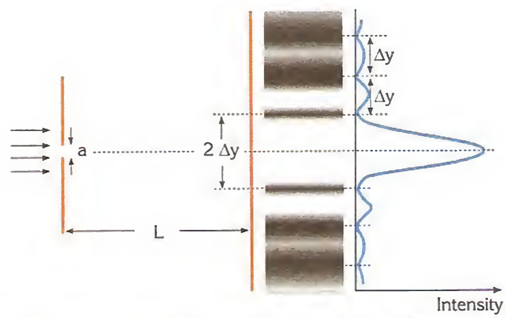
Example 5.5 showed that the central bright fringe is twice the size of the position of the first minimum. Therefore, the width of the central bright fringe is 2Δy

$$\Delta y = \frac{(2\,\text{m})(600\times10^{-9}\,\text{m})}{(0.05\times10^{-3}\,\text{m})}$$

$$\Delta y = 0.024\text{ m} = 24\text{ mm (fringe separation)}$$

The width of the central bright fringe is;

$$2\Delta y = 2(24\text{ mm}) = 48\text{ mm}$$



5.8 RESOLVING POWER

The ability of an optical instrument to distinguish between objects which are separated by a small angle is limited due to diffraction effects. For example two distant stars may be seen as a single star, because the light waves from the stars diffract through the eye-pupil and their central bright fringes overlap (Figure 5.25-a). If two objects can be distinguished they are said to be resolved, this is shown in Figure 5.25-b.

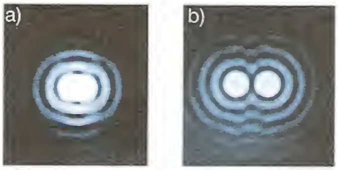


Figure 5.25 a) Two light sources which cannot be resolved b) Two light sources which can be resolved

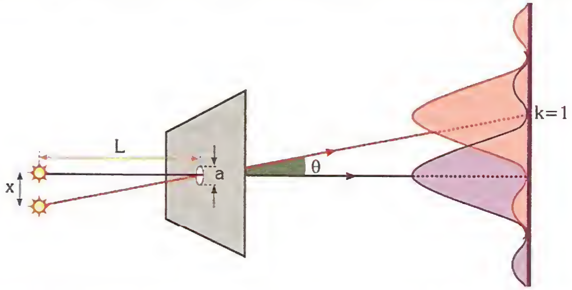


Figure 5.26 Rayleigh's criterion: Sources are said to be just resolved when the central bright fringe of one source coincides with the first minimum of the other.

Consider two light sources in front of a circular aperture forming a diffraction pattern on a screen, as in Figure 5.26. These light sources are said to be just resolved when the central bright fringe of one source coincides with the first minimum (m=1) of the other. This is called Rayleigh's criterion. Recall from Section 5.7 that for light diffracting through a single slit

$$a \sin\theta = k\lambda \qquad (k=1)$$

$$\sin\theta = \frac{\lambda}{a}$$

Since $\frac{\lambda}{a}$ is a very small, we can take $\sin\theta \approx \theta$. Thus, for the objects to be resolved, the minimum value of the angle is

$$\theta_{min} = \frac{\lambda}{a}$$

For small angels, we can take $\theta \approx \tan\theta \approx \frac{x}{L}$ as shown in Figure 5.26. Therefore the resolution criterion becomes,

$$\frac{\lambda}{a} = \frac{x}{L} \quad \text{just resolved}$$

Where λ is the wavelength of the light, a is the diameter of the aperture, x is the distance between the objects and L is the distance from the objects to the optical instrument. As a result,

If $\frac{\lambda}{a} < \frac{x}{L}$, then the objects are resolved; two objects are apparent

If $\frac{\lambda}{a} > \frac{x}{L}$, then the objects are not resolved; appears to be a single object

5.9 POLARISATION

Polarisation is a property of transverse waves. Since light is a transverse wave it can be polarised. Recall from section 4.2 that light is an EM wave which has electric field and magnetic field components oscillating perpendicular to each other and to the propagation direction. A beam of unpolarised light consists of many fields oscillating in different directions. polarised light consists of fields which oscillate in a definite orientation (Figure 5.27-a). The plane along which the electric field components of polarised light oscillate is called the plane of polarisation.

Now let's see how transverse waves are polarised. Consider a stretched string passing through a vertical slit as in Figure 5.28. A vertically oscillating wave can pass through the vertical slit. However, the same wave cannot pass through a horizontal slit.

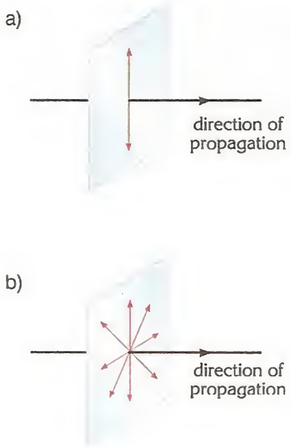


Figure 5.27 a) Oscillation direction of electric field b) Random electric field directions in unpolarized light.

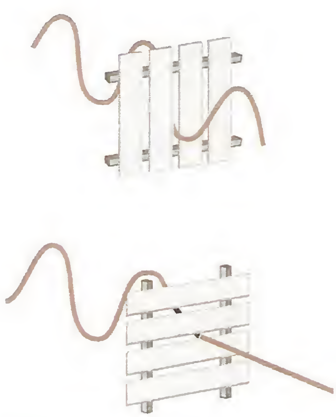


Figure 5.28 polarisation of waves in a stretched string

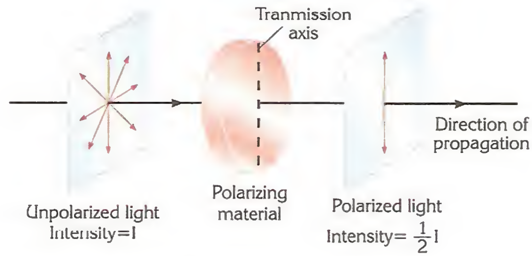


Figure 5.29 The intensity of unpolarized light is halved after passing through a polarising material

Similarly, polarising sunglasses consist of a layer of a substance called polaroid, which transmits light with an electric field oscillating only along a single line. This line is called the transmission axis of the polariser (Figure 5.29). Light with an electric field oscillating perpendicular to the transmission axis cannot be transmitted. Thus, polarised light can be obtained from unpolarized light using polarising filters. It is also possible to determine whether light is polarised or not using a polariser: If rotating the polariser changes the intensity of light and, at a certain angle, no light passes through the polariser, it means that incident light is polarised.

The intensity of unpolarized light is halved after passing through a single polariser (Figure 5.29). It is thus possible to adjust the intensity of the light by using a second

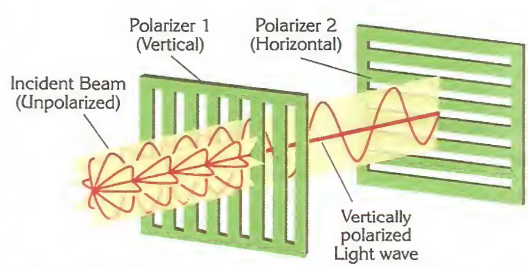


Figure 5.30 When two polarising sunglasses have their transmission axes perpendicular to each other, no light is transmitted. Therefore the background is observed to be black.

polariser. If the transmission axes of the polarisers are perpendicular to each other no light is transmitted. When two polarising sunglasses are held with their transmission axes perpendicular to each other no light is transmitted (Figure 5.30).

Light can also be polarised by reflection when it is incident on a medium having an index of refraction different from that of air. However, light can be completely polarised parallel to the reflecting surface only at a fixed incident angle. This angle occurs when the reflected and refracted rays are perpendicular and is called the polarising angle. If the incident angle is not equal to the polarising angle, light is only partially polarised. On sunny days glare, due to the reflection from roads or other horizontal surfaces, is a major problem for drivers. To reduce the effects of glare polarising sunglasses should have their polarisation axes in the vertical direction.

Summary

According to the wave theory, light is a wave. The relationship between the speed, frequency and wavelength of light is

$$c=\lambda f$$

Huygens' principle states that every point on a wavefront may be considered to be a source of secondary wavelets.

Dispersion is the dependence of the index of refraction on the wavelength or frequency of light, and is common to all waves. Usually, the index of refraction is lower for longer wavelengths.

Experimentally observed interference and diffraction phenomena indicate that light is a wave. Only coherent sources can produce an observable interference pattern. When monochromatic light is incident on two closely spaced slits, an interference pattern is observed on a distant screen. The positions of maxima and minima from the central bright fringe are given by

$$y=(k-\frac{1}{2})\frac{L\lambda}{d} \text{ (minima), } k=1, 2, 3, \ldots$$

$$y=k\frac{L\lambda}{d} \text{ (maxima), } k=0, 1, 2, 3, \ldots$$

where "d" is the distance between the slits, 'λ' is the wavelength of the light, and 'L' is the distance between the slits and the screen.

A diffraction grating is a device which has fine, parallel, equally spaced slits on its surface. A diffraction grating illuminated by a monochromatic light source produces thin bright lines, called interference maxima, on a screen. The condition for these maxima is

$$k\lambda=d\sin\theta, \; k=0, 1, 2, 3, \ldots$$

where "d" is the distance between two adjacent slits, "λ" is the wavelength of the light, and "θ" is the angular position of the maxima from the central bright fringe.

An interference pattern can be observed on a thin film due to the phase difference between the light waves reflected from the two surfaces of the thin film. When a thin film coating on glass is illuminated by monochromatic light

$$2d=\lambda, 2\lambda, 3\lambda \text{ (for constructive interference)}$$

$$2d = \frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, \ldots \quad \text{(for destructive interference)}$$

where "a" is the thickness of the film and "λ" is the wavelength of the light in the film.

$$\lambda_{film} - \frac{\lambda_{air}}{n_{film}}$$

A single slit produces a diffraction pattern on a screen when it is illuminated by light. However, light is said to be spread as wide as the central bright fringe. When a narrow slit is illuminated by monochromatic light, the position of minima is found by

$$y_k = k\frac{L\lambda}{a} \quad (k=1, 2, 3, \ldots)$$

where "a" is the width of the slit, "λ" is the wavelength of the light, and "L" is the distance from the slits to the screen.

The resolving power of an optical instrument is the ability of that instrument to distinguish between closely spaced objects. Resolving power is limited by diffraction effects, therefore an optical instrument with a larger objective has a better resolving power. The minimum angular separation between objects to be resolved is given by

$$\theta_{min} = \frac{\lambda}{a}$$

where "λ" is the wavelength of the light, and "a" is the slit width.

polarised light consists of waves all oscillating parallel to each other.

QUESTIONS AND PROBLEMS

5.1 Properties of Light Waves

5.2 Huygens' Principle

1. Find the frequency of a light wave of wavelength 4000Å .

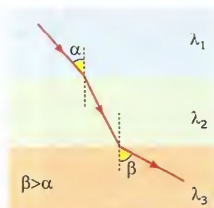
2. The wavelength of a monochromatic light source in air is 591 nm.

   a) What is the wavelength in glass (n_{glass}=1.5)

   b) Find the speed of light in glass

3. How do the speed, frequency, wavelength and direction of a light wave change when it passes from air into water?

4. The path followed by a light ray in three different media is shown in the figure. How do the wavelengths of light in these media compare?



5. A light wave has a wavelength of 600 nm in air and 400 nm in a medium. Find the index of refraction of the medium.

6. How are wavelength and amplitude of light perceived by a human being?

7. Why is the interference pattern of two ordinary light sources unobservable?

8. In order to produce wider interference fringes, which colour of light must be used in a double slit experiment, blue or red?

9. In order to decrease the fringe separation in a double slit experiment how must the following quantities be changed? (d) slit separation, (L) slit-screen distance, (f) frequency of light

10. How do speed, frequency and wavelength of a monochromatic light source change, when passing from air into water?

11. To increase the number of fringes produced on a screen in a double slit experiment what must be done?

12. If the distance between the slits and the screen in a double slit experiment is filled with a liquid how does the interference pattern change?

13. Why are the slits narrow in a double-slit experiment?

14. A monochromatic light of wavelength 6000 Å passes through two slits 0.3 mm apart and falls onto a screen placed 1 m away from the slits. Find

a) Fringe separation

b) Distance from the central line to the 2^{nd} dark fringe

c) Distance from the central line to the 3^{rd} bright fringe

15. A monochromatic light source is passed through two slits 0.3 mm apart and falls onto a screen placed 2 m away from the slits. If the fringe separation is 4 mm, find

a) The wavelength of the light

b) The distance between the 2^{nd} bright fringe and the 3^{rd} dark fringe on opposite sides of the central maximum.

16. In a double slit experiment the fringe separation is 2.5 mm. What is the new fringe separation if the distance to the screen from the slits is doubled and the slit separation is halved?

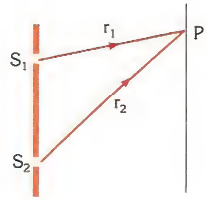
17. In a double slit experiment the distance between the 1^{st} and 4^{th} dark fringes on the same side is measured to be 7.5 mm. What is the distance from the 2^{nd} bright fringe to the central line?

18. The third dark fringe occurs 2.5 cm away from the central line on a screen 3 m away from a double slit arrangement.

a) If the slits are 0.15 mm apart, what is the wavelength of the light?

b) If light of wavelength 6500 Å is used in the same experiment what will the fringe separation be?

19.



In a double slit experiment, as shown in the figure $|r_1 - r_2| = \frac{5\lambda}{2}$. Which fringe is formed at point P?

20. The distance between two successive maxima in a double slit interference pattern is 6 mm. If the distance from the slits to the screen is 1 m and the wavelength of monochromatic light used is 450 nm, what is the distance between the slits in mm?

21. In a double slit experiment, light of wavelength λ is used. Which fringe is formed at a point on the screen 18λ and 15λ away from the slits?

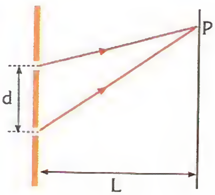
22. When light of wavelength λ=700 nm is used in Young's experiment, the 3^{rd} minimum forms at point P. Which fringe would form at the same point if in the experiment light of wavelength λ=500 nm were used.

23. In a double slit experiment, the 3rd dark fringe forms on a point P on the screen . If the space between the slits and the screen is filled with a liquid, the 3rd bright fringe forms at the same point P. What is the refractive index of the liquid?

24. In a double slit experiment fringe separation is measured to be "y" when light of wavelength λ=500 nm is used. The space between the slits and the screen is filled with a liquid (n=1.2). In order to get the same fringe separation what must the wavelength of the light source be?

25. What is the difference between the phenomena of diffraction and interference. What is common to both?

26.



In a double slit experiment set-up as in figure λ=690 nm, d= 0.01 mm. What is the angular separation between the 2^{nd} and 5^{th} maxima?

27. In a double slit experiment one of the slits is covered with a thin transparent film. What changes occur in the interference pattern?

5.5 Diffraction Gratings

28. A diffraction grating has 100 slits in 1 mm. Find the distance between two successive slits.

29. Light of wavelength 680 nm is incident on a diffraction grating containing 125 lines/mm. Find the angular positions of the first and the third maxima.

30. In a diffraction grating two successive slits are 0.005 mm apart.

a) Find the number of silts in 1 mm

b) Find the distance from the central maximum to the second bright fringe on a screen 2 m away, if the wavelength of the light is 600 nm.

31. A laser pointer containing wavelengths between 635 nm and 670 nm is incident on a 500 slits/cm diffraction grating. How wide is the second order maximum on a screen 1 m away from the grating?

32. A monochromatic light of wavelength 650 nm is incident on a diffraction grating which has 5000 slits/cm. How many bright lines can be observed through the grating?

33. The number of bright lines which can be observed through a diffraction grating is 9. What is the maximum number of slits per centimetre for this grating if the wavelength of the light is 5000 Å?

5.6 Thin Films, Newton's Rings

34. Why does the top of the soap film in the figure appear black while other parts appear to be different colours? Estimate the thickness of the dark region

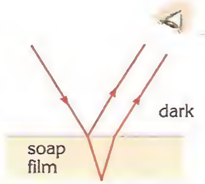
35. Thin films such as a soap bubble are normally, transparent, How do they eliminate light? Why don't thick films eliminate light?

36. Why do the colours of the rainbow and the colours on a soap bubble have different tones (paler or darker colours)?

37. A camera lens is coated with a film to reduce reflection from its surface. What is the minimum thickness required for the film not to reflect yellow-green light (to which the human eye is most sensitive) of wavelength 500 nm? (Take n_{film}= 1.38)

38. A film of oil of uniformly increasing thickness produces an interference pattern . The oil film (n=1.42) is illuminated by a sodium lamp (λ=589 nm). Find the difference in the thickness of two adjacent red fringes.

39. A soap film (n=1.3) is illuminated with monochromatic light having a wavelength of 520 nm. What is the minimum thickness of the soap film required for the eye to observe total darkness over the film surface?



5.7 Single Slit Diffraction

40. Monochromatic light of wavelength 550 nm passes through a slit having a width of 0.1 mm. What is the width of the central bright fringe on the screen 1 m away from the slit?

41. Monochromatic light of wavelength 700 nm passes through a slit having a width of 0.02 mm. What is the angular width of the central bright?

42. Monochromatic light is passed through a slit having a width of 0.02 cm. If the width of the central bright fringe is 4 mm on a screen placed 100 cm from the slit, what is the wavelength of the light?

43. Which spreads more after passing through the same single slit, red or blue light?

44. A single slit is illuminated by a 520 nm wavelength monochromatic light source. If the angle between the first dark fringes on both sides of the central bright fringe is 6°, what is the slit width?

45. A 0.19 mm wide single slit is illuminated by a 5700 Å wavelength light source. How far away is the 3rd dark fringe from the central bright fringe on a screen 3 m away from the slit?

46. A monochromatic light source of wavelength 450 nm in incident on a single slit having a width of 0.05 mm. Find the distance between the 2nd dark fringe on one side of the central bright fringe and the 3rd bright fringe on the other side of the central bright fringe. The screen is placed 2 m away from the slit.

47. Why cannot a light ray be sent along a straight line path for a long distance?

48. The shadows of objects are not sharp even with a point light source, why?

49. The angular size of the central bright fringe produced by a single slit is 4°. Estimate the wavelength of the light if the slit is 0.02 mm wide.

5.8 Resolving Power

50. How can you improve the resolving power of an optical instrument?

51. Two light sources which are 6 cm apart can just be be resolved through a 5 mm wide slit, 500 m away. Find the wavelength of the light.

52. Why do larger objective lenses produce better views in telescopes?

5.9 Polarisation

53. Can sound waves be polarised? Why/Why not?

54. How can you determine whether a beam of light is polarised or not?

ANSWER KEY

Chapter 1

Mechanical Oscillations

1. 3600^o, 20π rad

2. 4 Hz, 0.25 s

3. $\frac{5}{4}$

6. 0.25 s, 4 Hz

8. 80 cm

9. a) 0.83 Hz

b) 0.1 m

12. a) 20 J

b) 20 J

13. a) 24 J, 0

b) 0, 24 J

14. $200\ m/s^2$, $2\sqrt{5}$ m/s

16. 1

17. a) 0, $25\ m/s^2$

b) $\sqrt{5}$ m/s, 0

18. $\frac{A_1}{A_2} = \frac{1}{2}$

19. a) $|\vec{v}| = 1.6$ m/s, $a = -24\ m/s^2$

b) $|\vec{v}| = 1.6$ m/s, $a = 24\ m/s^2$

20. 8 m/s, $120\ m/s^2$

22. 0.6 m/s

23. a) 0.1 m

b) 0.8 m/s

24. $10\sqrt{2}$ m

25. a) 0.004 s

b) 500π rad/s

26. 6 Hz, 12π rad/s

27. a) 20 cm

b) 12

28. 0.05 m, 3.14 m/s

29. $x(t) = 0.02\cos(20\pi t)$ in m

30. a) $x(t) = 0{,}1\cos(0.5\pi t)$ in m

b) 7.1 cm

31. a) $v(t) = -0.6\sin(6t)$ in m/s

b) $a(t) = -3.6\sin(6t)$ in m/s^2

32. a) $x(t) = 0.1\cos(2\pi t)$ in m

b) 3.94 N

33. $\frac{f_{blue}}{f_{red}} = 1.5$

34. a) 0.1 m, 2.5 m/s, $63.1\ m/s^2$

b) $x(t) = (0.1\ m)Cos(8\pi t)$

$a(t) = (-63.1\ m/s^2)Cos(8\pi t)$

35. 0.2 s

37. a) 30 cm

b) 21.2 cm

38. 10.6 cm

39. t=2 s

40. a) $\frac{\sqrt{3}}{2}x_{max}$

b) $-\frac{x_{max}}{2}$

41. 1

42. *i.* c *ii.* a *iii.* b

43. 0.5 s

44. a) 0

b) 4π rad

45. $\frac{\pi}{3}$ rad

46. a) $x(t) = 0.1\cos(10\pi t)$ in m

b) $x(t) = 0.1\sin(10\pi t)$ in m

47. $\frac{\pi}{4}$ rad

48. a) 10 cm

b) 4.35 m/s

49. a) $x(t) = 0.2\sin(8\pi t + \frac{\pi}{6})$ in m

50. $a(t) = (-394.4\ mm/s^2)\cos(20\pi t + \frac{\pi}{3})$

55. a) 0.21 s

b) 0.21 s

56. 1 kg

57. 0.225 kg

58. a) 2.4 m/s, $7.2\ m/s^2$

b) $x(t) = 0.8\cos(3t)$ in m

59. 4.35 m/s

60. 1.78 Hz

61. 94.8

62. a) $x(t) = 0.2\sin(5t)$ in m

b) 60 cm

63. b) 2

64. a) 6.34 s

b) 15.7 s

66. a) $29\ m/s^2$

67. $\frac{L_1}{L_2} = 16$

68. 55.6 cm

71. $\frac{1}{18}$

72. 5.36 s

73. $\frac{1}{2}$

74. 0.5 s

75. 18

80. 78.9 N/m

Chapter 2

Mechanical Waves

3. 10

5. 0.2 s, 5 Hz

11. 4 m

12. 0.067 s, 15 Hz

13. 1.8 m/s

14. 0.3 m

15. 1 MHz

16. 3 m/s

17. 10 cm

19. 1.1 m/s, 0.37 m

21. 5 Hz

22. 4.25 m - 0.25 m

25. a) 0.3 m

b) 0.2 m

27. 2.5 ms

28. $\frac{2\pi}{3}$ rad

29. a) 5 cm/s

b) 4 Hz

30. $y = 3\text{Sin}\left[20\pi\left(t - \frac{x}{4\text{ cm}}\right)\right]$ in cm

31. $y=0.03\sin[4\pi(t-0.25)]$ in m

32. 1 cm

34. a) 2 s, 6 cm

b) 3 cm/s in -x direction

35. a) 12.56 cm/s

b) +x direction

36. a) 50.24 cm/s in -x direction

b) 3.14 cm, 16 Hz

37. 0.25 m

38. 0.5 m

39. 10 π

40. 2.5 π

41. a) $\frac{\pi}{6}$ rad

b) π rad

c) 3π rad

42. a) 1.6 m/s

b) to the right

43. 600°

44. 1 m

45. $v_C > v_A > v_B$

58. 3430 m

59. 5972 m

60. 734 m

61. 18.7 m

62. 0.67 m

63. 240

65. a) 0.34 m

b) 1.5 m

66. 74.8 m

67. 196.3 m/s

68. 10^{-8} W/m^2

70. 60 dB

71. 176.99 dB

76. 34 cm, 1000 Hz

77. 2 Hz, 0.9 m

78. a) 3 cm, 20 Hz

b) 37°

79. c) 30°

81. 5 s

83. 16 cm

84. 2 cm

85. 2nd minimum

92. 100 Hz

93. 34.93 cm from the "bridge"

94. 221.2 N

95. 30 cm

96. a) 1320 Hz

b) 1760 Hz

99. 400 m/s

100. 4

Chapter 3

Electromagnetic Oscillations

3. No change

4. Zero

7. a) 0.02 J

b) 2 mC

9. 1 mJ

10. 2.5 mJ

11. 0.6 J

12. 21 J

13. a) 0.25 A

b) 0.47 J

14. a) 0.1 C

b) 0.05 C each

c) 0.5 J

15. a) 5000 Hz

b) 0.1 μC

16. 0.4 mC

17. 5 nC, zero

18. a) 2 V

b) $U(t) = 2V \cos(10^5 \pi t)$

19. $i(t) = -90 \text{ mA} \sin(300t)$

20. 1000 rad/s

21. 0.4 mH

22. 10^4 - 10^9 Hz

23. $U(t) = 20 \text{ V} \cos(100t)$

$q(t) = 1 \text{ mC} \cos(100t)$

$i(t) = -0.1 \text{ A} \sin(100t)$

24. a) 100V

b) zero

25. 2.6×10^{-4} s

26. Decrease by a factor of 2

27. 10 nF

28. Decrease by a factor of 2

29. Increase by a factor of 3

32. 2 J

33. a) 0.01 J

b) 0.01 J

34. 3 kHz

35. 2

36. 0.2 μC

37. 0.4 A

38. 1.25 mJ

39. a) 0.32 J

b) $\frac{1}{255}$

40. a) 10^{-5} J

b) 62 μC

41. 0.1 J, 0.3 J

42. 10 V

43. $\frac{\sqrt{2}}{2}$ mA, $2\sqrt{2}$ μC

44. 6.4 mJ, 33.6 mJ

45. a) zero

b) 1

46. 30 mA

47. a) $\frac{1}{2}$

b) $\frac{1}{2}$

c) $\frac{1}{4}$

48. 0.042 J

49. a) $U(t) = 10 \text{ V} \sin(5000t + 0.643)$

b) 0.04 A

Chapter 4

Electromagnetic Waves

6. 200 m

7. 3×10^8 m

8. 30 MHz - 3 MHz

10. 15 km

11. a) 6×10^{-3} V/m

b) 333.3

12. a) 1.5×10^{-8} Hz

b) $-z$, 90 V/m

13. b) 0.26 s

14. 65 min 23 s

15. 75 km

16. 60 km, 150 m

17. $B(x,t) = (2 \times 10^{-13} \text{T}) \text{Sin} \left[10^7 \times \left(t - \frac{x}{c}\right)\right]$

1.59 Mhz

18. $E(x,t) = (100 \text{V/m}) \text{Sin} \left[4\pi 10^6 \cdot \left(t - \frac{x}{c}\right)\right]$

19. 15 MHz, 20 m, +x

21. 2 km

24. 1.67×10^{-12} J/m^3

25. 0.03 W/m^2

26. 3.98×10^{-5} W/m^2

27. 3×10^{-5} W/m^2

28. 0.2 W/m^2

29. $\frac{1}{4}$

30. 40 V/m

31. 500 m

32. 2.82 m

34. 10^{-8} s

35. 300 m

36. 1.59 kHz

37. 9.4×10^5 m

38. 20 MHz

40. 5 pF

41. 282 pF

42. $C_2=16C_1$

43. 18.84 km - 188.4 m

Chapter 5

Wave Optics

1. 75×10^{13} Hz

2. a) 394 nm
   b) 200 000 km/s

4. $\lambda_3>\lambda_1>\lambda_2$

5. n = 1.5

14. a) = 2 mm
    b) $y_{2(min)}$ = 3 mm
    c) $y_{3(max)}$ = 6 mm

15. a) λ= 6000 Å
    b) 18 mm

16. Δy becomes four times larger = 10 mm

17. 5 mm

18. a) 5000 Å
    b) 1.3 cm

19. 3rd minimum

20. 0.075 mm

21. 3rd maximum

22. 4th minimum

23. n = 1.2

24. 600 nm

26. 12.3°

28. 0.01 mm

29. for the 1st maximum θ= 4.9°
    for the 3rd maximum θ=14.8°

30. a) 200 slits/mm
    b) 48 cm

31. 3.5 mm

32. 7 lines

33. 5000 slits/cm

37. 90.6 nm

38. 207.4 nm

39. 200 nm

40. 11 mm

41. 4°

42. 400 nm

44. 0.01 mm

45. 2.7 cm

46. 9.9 cm

49. 698 nm

51. 600 nm

WAVES

MODULAR SYSTEM

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